

Identifying Symmetry in Equations

Graphs of Equations on a coordinate plane can have symmetry with respect to the X-Axis, Y-Axis, and/or the Origin. Some equations have no symmetry, and some equations have multiple types of symmetry. Each type of symmetry can be determined individually using either graphical or algebraic test methods. Testing for algebraic symmetry can aid in sketching the graphs of equations.

I. Graphical Test for Symmetry –

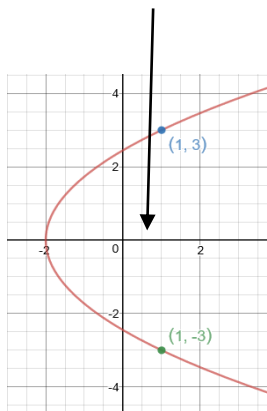
*Even Functions have
Y-Axis Symmetry!*

*Odd Functions have
Origin Symmetry!*

X-Axis Symmetry:

If the point (x, y) is on the graph, the point $(x, -y)$ is also on the graph.

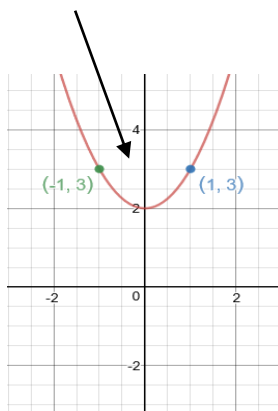
The X-Axis acts like a mirror.



Y-Axis Symmetry:

If the point (x, y) is on the graph, the point $(-x, y)$ is also on the graph.

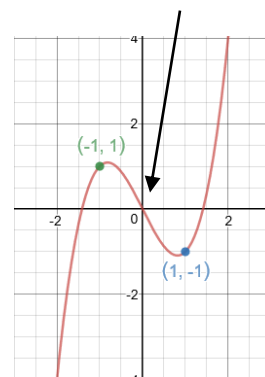
The Y-Axis acts like a mirror.



Origin Symmetry:

If the point (x, y) is on the graph, the point $(-x, -y)$ is also on the graph.

If you spin the picture upside down about the Origin, the graph looks the same!



II. Algebraic Test for Symmetry –

X-Axis Symmetry: Occurs if “ y ” is replaced with “ $-y$ ”, and it yields the original equation.

Y-Axis Symmetry: Occurs if “ x ” is replaced with “ $-x$ ”, and it yields the original equation.

Origin Symmetry: Occurs if “ x ” is replaced with “ $-x$ ” and “ y ” is replaced with “ $-y$ ”, and it yields the original equation.

Example: Determine whether the following equation has any type(s) of symmetry. $x^2 + y^2 = 16$

X – Axis Symmetry

$$x^2 + (-y)^2 = 16$$

$$x^2 + y^2 = 16$$

Yes → X-Axis Symmetry

Y – Axis Symmetry

$$(-x)^2 + y^2 = 16$$

$$x^2 + y^2 = 16$$

Yes → Y-Axis Symmetry

Origin Symmetry

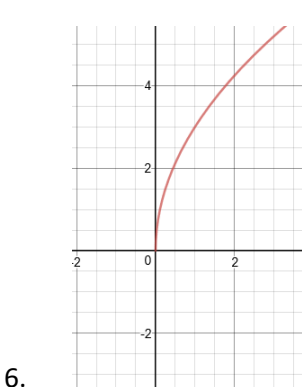
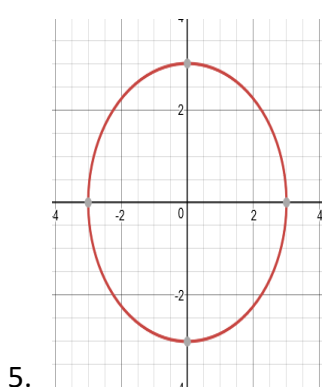
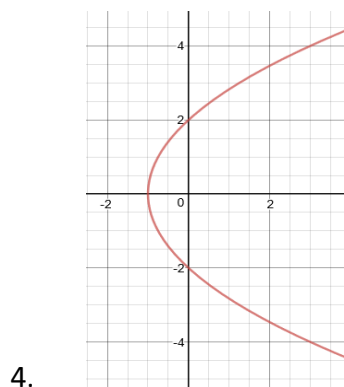
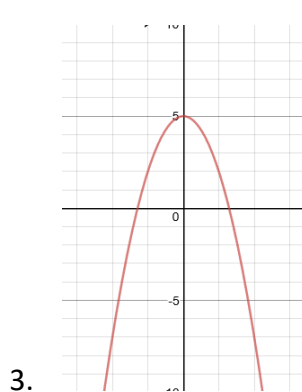
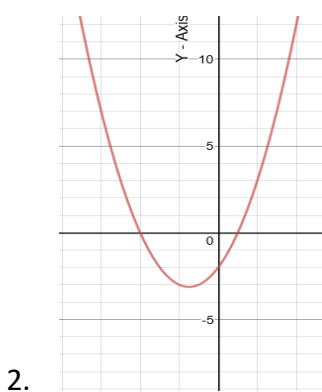
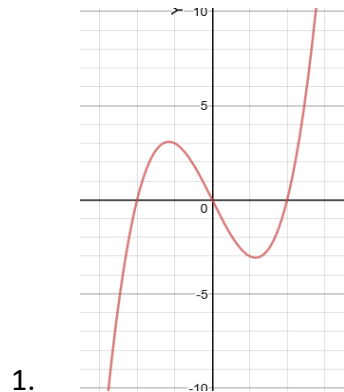
$$(-x)^2 + (-y)^2 = 16$$

$$x^2 + y^2 = 16$$

Yes → Origin Symmetry

Equation Symmetry - Practice Problems

A. Graphically determine what type(s) of symmetry, *if any*, are present.



B. Algebraically check for symmetry with respect to the x-axis, y-axis, and the origin.

1. $y = x^2 + 4$
2. $y = -x^3 - x$
3. $y = 2x - 10$
4. $x = -y^2 + 4$
5. $x^2 + y^2 = 25$
6. $y = |x| + 2$

Answers:

Section A (Graphically)

1. Origin
2. None
3. Y-Axis
4. X-Axis
5. X-Axis, Y-Axis, Origin
6. None

Section B (Algebraically)

1. Y-Axis
2. Origin
3. None
4. X-Axis
5. X-Axis, Y-Axis, Origin
6. Y-Axis