Finding the Domain of a Function

When finding the domain of a function, start with the assumption that all real numbers, $(-\infty,\ \infty)$ will work with the following exceptions:

1. If there are any fractions, the denominator(s) must NOT be equal to ZERO.
2. If there are any radicals with an even index, $\sqrt[\text{even number}]{\ }$, then the radicand (the part under the radical MUST be set to ≥ ZERO.
3. *For Math 120 or higher students see other side*
   The arguments of any logarithmic functions: *log* ( ) or *ln* ( ) MUST be > ZERO.
4. If there is a radical $\sqrt[\text{even number}]{\ }$ and a rational function, then the denominator MUST be > ZERO.

Examples:

1. Find the domain of $f(x) = 3x^4 - 5x - 7$.

   None of the exceptions stated above apply, therefore the domain is ALL Real Numbers, $(-\infty,\ \infty)$.

2. Find domain of $g(x) = \sqrt[5]{x-7}$.

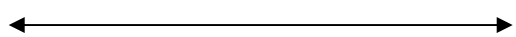
   None of the exceptions stated above apply, the index on the radical, namely 5, is an odd number, therefore, the domain is ALL Real Numbers, $(-\infty,\ \infty)$.

3. Find the domain of $h(x) = \frac{1}{4}x^2 - x + 2$.

   The only denominator in the function is the number 4 which is not equal to zero, therefore none of the exceptions apply and the answer is ALL Real Numbers, $(-\infty,\ \infty)$.

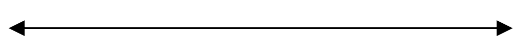
4. Find the domain of $k(x) = \frac{x-2}{x+3}$

   Exception #1 above applies to this problem as it involves a fraction. The numerator, namely, the *x-2*, is not relevant to the domain of the function. Only the denominator is relevant: one must insure that *x+3* is NOT equal to ZERO, therefore $x \neq -3$. Thus the answer for the domain can be written in three ways.

   a. Set-builder: $\{x | x \neq -3\}$
   b. Graphical: 
   c. Interval: $(-\infty,\ -3) \cup (-3,\ \infty)$

5. Find the domain of $k(x) = \frac{x-2}{x^2 - 7x + 12}$

   Exception #1 above applies to this problem as it involves a fraction. The numerator, namely, the *x-2*, is not relevant to the domain of the function. Only the denominator is relevant: one must insure that $x^2 - 7x + 12$ is NOT equal to ZERO, therefore $(x-4)(x-3) \neq 0$ *so* $x \neq 4,\ 3$. Thus the answer for the domain can be written in three ways.

   a. Set-builder: $\{x | x \neq 3,\ 4\}$
   b. Graphical: 
   c. Interval: $(-\infty,\ 3) \cup (3,\ 4) \cup (4,\ \infty)$

6. Find the domain of $g(x) = \sqrt[6]{4-2x}$.

   Exception #2 above applies: there is a radical with an even index, namely, 6. Therefore, the radicand must be $\ge$ 0.

   $\underset{-4}{4} - 2x \ge \underset{-4}{0}$

   $\dfrac{-2x}{-2} \ge \dfrac{-4}{-2}$ (when dividing by a negative number in an inequality, the inequality sign must change)

   $x \le 2$

   The final answer can be written in these ways.

   a. set builder: $\{x | x \le 2\}$
   b. graphical:
   c. interval: $(-\infty,\ 2]$

For Math 120 or higher

Examples:

1. Find the domain of $g(x) = \log\left(x^2 - x - 2\right)$

   Exception #3 applies for this problem. Therefore the argument of the *log* function must be > 0.

   Therefore, $\begin{array}{l} x^2 - x - 2 > 0 \\ (x-2)(x+1) > 0 \end{array}$ critical values that would make argument equal to zero are *x=2* and *x=-1*

Test values

Critical points

Using test values of *x=-2*, *x=0*, and *x=3* generates the following results: -2 generates a 4 which is greater than 0 so this region works; 0 generates a -2 which is not greater than 0 so this region does not work; 3 generates a 4 which is greater than 0 so this region works.

Therefore the intervals that work are $(-\infty,\ -1)$ *and* $(2,\ \infty)$ and the domain for *g(x) is* $(-\infty,\ -1) \cup (2,\ \infty)$.

2. Find the domain of $h(x) = \dfrac{1}{\sqrt{x+2}}$

   Exception #4 applies for this problem. There is a radical in the denominator with an even index. Therefore the radicand must be > 0.

   $x + 2 > 0$

   $x > -2$

   The final answer may be written in these ways:

   a. set builder: $\{x | x > -2\}$
   b. graphical:
   c. interval: $(-2,\ \infty)$

3. Find the domain of $g(x) = \dfrac{\sqrt{x-3}}{x^2+7x+10}$

Exceptions #1 and 3 apply for this problem. There is a radical in the numerator with an even index. Therefore the radicand must be ≥ 0 and the denominator cannot equal zero.

$x-3 \geq 0$

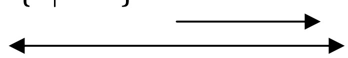
$x \geq 3$

$(x+5)(x+2)$ The domain would be $x \geq 3$.

$x \neq -5, \ -2$

The final answer may be written in these ways:

a. set builder: $\{x | x \geq 3\}$

b. graphical: 

c. interval: $(3, \ \infty)$

4. Find the domain of $k(x) = \dfrac{\sqrt{x-2}}{x-5}$

Exceptions #1 and 3 apply for this problem. There is a radical in the numerator with an even index. Therefore the radicand must be ≥ 0 and the denominator cannot equal zero.

$x-2 \geq 0$

$x \geq 2$

$(x-5) \neq 0$ The domain would be $x \geq 2$ *and* $x \neq 5$

$x \neq 5$

Therefore the domain in interval notation would be: $[2, \ 5) \cup (5, \ \infty)$

$(-2-2)(-2+1)$

$(-4)(-1)$ 4 is greater than 0 so this region works

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$(0-2)\cup(0+1)$

$(-2)(1)$ -2 is not greater than 0 so this region does not work

-2

$(3-2)(3+1)$

$(1)(4)$ 4 is greater than 0 so this region works

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