Normal Distributions and the Empirical Rule

Normal Distribution – A data set that is characterized by the following criteria –

- The *Mean* and *Median* of the distribution are equal to the *Mode*.
- Most data values are clustered near the Mean (or Mode) so that the distribution has a well-defined peak.
- Data values are then spread evenly around the Mean (or Mode) so that the distribution is *symmetric*.
- Data values become increasingly rare as you move farther to the right and to the left of the Mean. This results in tapering tales on both ends of the curve.
- The variation is characterized by the standard deviation of the data distribution.

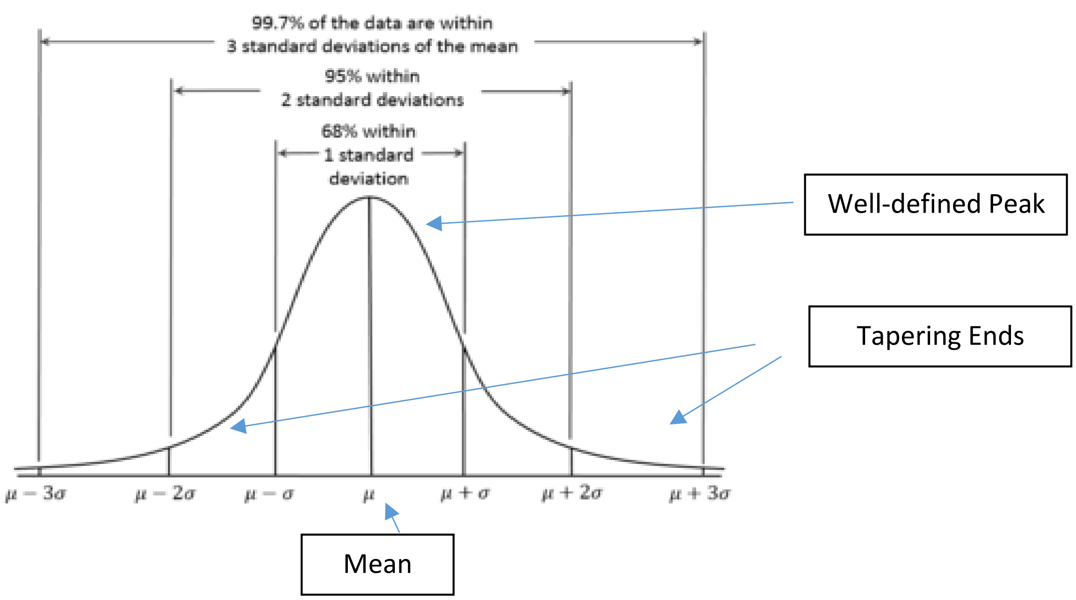
Note – This is sometimes also referred to as a "Normal Curve" or a "Bell-Shaped Curve."

Empirical Rule - When a histogram of data is considered to meet the conditions of a "Normal Distribution", *(i.e. its graph is approximately bell-shaped)*, then it is often possible to categorize the data using the following guidelines… *(Note: $\sigma \rightarrow$ symbol used for standard deviation.)*

- About 68% of the data *(68.3%)* is within one standard deviation ($\pm 1\ \sigma$) of the mean (μ).
- About 95% of the data *(95.4%)* is within two standard deviations ($\pm 2\ \sigma$) of the mean (μ).
- About 99.7% of the data *(all or almost all)* is within three standard deviations ($\pm 3\ \sigma$) of the mean (μ).

*Note - This rule is also sometimes called the "**68 – 95 – 99.7 Rule**."*

The Empirical Rule is illustrated in the picture below.



Note: *The Empirical Rule implies that a data set that is normally distributed has a width of approximately 6 standard deviations* ($Width\ \approx 6\ \sigma$).

Standard Deviation – A measure of how far data values are spread around the mean of a data set. It is computed as the square root of the variance. The actual formula for calculating the standard deviation depends on whether the data represents a population or is from a sample.

Population Standard Deviation:

$$\sigma = \sqrt{population\ variance} = \sqrt{\frac{\sum (x_i - \mu)^2}{n}}$$

Sample Standard Deviation:

$$s = \sqrt{sample\ variance} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$n = \#\ of\ data\ points$

$x_i = individual\ data\ points$

$\mu = population\ mean$

$\bar{x} = sample\ mean$

The "Range Rule of Thumb" - The standard deviation is sometimes *approximated* by using the *range* of a data distribution according to the following …

Approximately

$$Standard\ Deviation \approx \frac{Range}{4} = \frac{High - Low}{4}$$

Note: This approach works well in data sets where the values are evenly distributed and there are not any outliers.

Example - For the following data set, approximate the standard deviation using the ***range rule of thumb***.

8.2	8.8	9.2	10.6	12.7
8.4	9.0	9.7	11.6	14.0
8.5	9.2	10.4	11.8	15.9
8.8	9.2	10.5	12.6	16.1

Lowest data point (8.2)

Highest data point (16.1)

Standard Deviation

$$\sigma = \frac{High - Low}{4} = \frac{16.1 - 8.2}{4} = 1.975$$

Example - If for a certain data set, the standard deviation is $\sigma = 4.5$ and the mean is $\mu = 20.2$.

a. In between what two values is approx.. 68% of the data? → Answer: $\mu \pm 1\sigma = 20.2 \pm 4.5$

$= 15.7\ and\ 24.7$

b. In between what two values is approx. 95% of the data? → Answer: $\mu \pm 2\sigma = 20.2 \pm 2(4.5)$

$= 11.2\ and\ 29.2$

c. In between what two values is all or almost all of the data? → Answer: $\mu \pm 3\sigma = 20.2 \pm 3(4.5)$

$= 6.7\ and\ 33.7$