

Expected value $E(x)$ - (Mean of a random variable)

Expected Value – The mean of a random variable. It is often used to *predict how much is likely to be gained or lost based on making a certain decision or taking a specific action*. If the expected value is positive, a gain is expected. If it is negative, a loss is expected. The expected value represents the average gain (profit) or loss if the same decision is repeated over and over again. In application, the expected value is typically illustrated through ‘games of chance’. In general, it can be calculated as $E(x) = \sum [x \cdot P(x)]$, or

$$E(x) = \sum(\text{Profit})(\text{Probability of Profit}) - \sum(\text{Cost})(\text{Probability of Cost})$$

Examples -

Life Insurance Policies –

An insurance company sells a one-year term life insurance policy to a 75-year old woman. The woman pays a premium of \$750. If she dies within 1 year, the insurance company will pay \$25,000 to her beneficiary. According to the 2014 Social Security Actuarial Life Table, the probability that a 75-year old woman will be alive in 1 year is 0.9746. Find the expected value of the insurance company’s profit on the policy. Interpret the result.

$$E(x) \text{ of Profit} = (\$750)(0.9746) - (\$25,000 - \$750)(1 - 0.9746) = \$115$$

The amount required to purchase the policy

The probability that the woman lives for at least 1 year

The amount the insurance company “loses” if the woman dies

The probability of the woman dying within 1 year

Interpretation - This indicates that if the insurance company sells many policies like this one, the company can expect on average to earn a \$115 profit per policy. (Gain)

Lottery Tickets –

In a state lottery, you pay \$1 and pick a number from 000 to 999. If your number comes up, you win \$600. The probability of winning is 0.001. What is the expected value of your profit? Is this a gain or loss?

$$E(x) = (\$600 - 1)(0.001) - (\$1)(0.999) = -\$0.40 \text{ (Loss)}$$

The amount that you can profit

The probability of winning

The cost of a ticket

The probability of losing

Practice Problems -

Raffle Tickets –

You purchase a \$5 raffle ticket for a fundraiser. A total of 500 tickets are sold. 5 winners will be drawn. 1st prize will be \$250. 2nd prize will be \$150. 3rd prize will be \$100. The 4th and 5th place winners will each receive \$50. What is the expected value of your winnings?

$$\begin{aligned} \text{Answer: } E(x) &= (\$245) \left(\frac{1}{500}\right) + (\$145) \left(\frac{1}{499}\right) + (\$95) \left(\frac{1}{498}\right) + (\$45) \left(\frac{1}{497}\right) + (\$45) \left(\frac{1}{496}\right) - (\$5) \left(\frac{495}{500}\right) \\ &= -\$3.80 \text{ (Loss)} \end{aligned}$$

SAT Scores –

- a. You are taking an SAT test. If you get the question right, you gain one point, but if you get the question wrong, you lose $\frac{1}{4}$ of a point. You have no idea what the correct answer is and randomly take a guess from the 5 choices given. What is the expected value for the number of points you get?

$$\text{Answer: } E(x) = (1)(0.2) - (1/4)(0.8) = 0$$

- b. On the next SAT test question, you confidently eliminate 2 of the choices and randomly guess from the 3 remaining options. What is the expected value for the number of points you get?

$$\text{Answer: } E(x) = (1)(1/3) - (1/4)(2/3) = 0.1666\dots \text{ (Gain)}$$

Gambling –

In the game of “Craps”, a pair of dice are rolled. You can bet \$1 that the sum of the dice will be 2 (“snake eyes”). The probability of winning is $1/36$. If you win, the **profit** is \$30. What is the expected value of your profit?

$$\text{Answer: } E(x) = (\$30)(1/36) - (\$1)(35/36) = -\$0.14 \text{ (Loss)}$$

Business Opportunity –

You are considering investing \$10,000 into a start-up company. You estimate that you have a 0.25 probability of a \$20,000 loss, a 0.20 probability of a \$10,000 profit, a 0.15 probability of a \$50,000 profit, and a 0.40 probability of breaking even (no profit). What is the expected value of the profit? Should you follow through on this investment?

$$\begin{aligned} \text{Answer: } E(x) &= (\$10,000)(0.20) + (\$50,000)(0.15) + (\$0)(0.40) - (\$20,000)(0.25) \\ &= \$4500 \text{ (Yes)} \end{aligned}$$