The Fundamental Principle of Counting

Counting - If a sequence of several operations is being performed, the **total number of ways** to perform that sequence can be found by multiplying together the number of ways to perform each individual operation.

Examples -

#1. While dining at a restaurant, there are 6 appetizers, 15 main courses, and 4 desserts to choose from. If an individual orders 1 appetizer, 1 main course, and 1 dessert, how many possible menu combinations could be created?

Answer → $\#\ of\ possible\ menu\ selections = 6 \cdot 15 \cdot 4 = 360$

#2. As a student, you are planning to take 1 Math class, 1 Science class, and 1 English class next semester. How many different possible schedule combinations could you come up with if you can choose between 3 Math classes, 4 Science classes, and 2 English classes?

Answer → $\#\ of\ possible\ schedule\ combinations = 3 \cdot 4 \cdot 2 = 24$

Computing Factorials

Factorial (!) - If n is a positive integer, then *n!* is the product of all positive integers starting from n down through 1.

$$\boldsymbol{n\ factorial = n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1}$$

Examples - Evaluate 5! ***Answer***: $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

Evaluate $\frac{8!}{4!}$ ***Answer:*** $\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{4 \cdot 3 \cdot 2 \cdot 1}} = 8 \cdot 7 \cdot 6 \cdot 5 = 1680$

<u>Permutations and Combinations</u>

Permutations and Combinations both involve arranging *r items* chosen from a group of *n items*. The difference between the two, involves whether the ordering of those items matters or not.

Permutations - *The number of different ways that a group of things can be ordered.* This ordered arrangement happens when no item is used more than once and <u>the order of the arrangement makes a difference</u>.

The Permutation formula is -

$${}_nP_r = \frac{n!}{(n-r)!}$$

n = the number of items in the group
r = the number of items that are being selected

Example - 10 baseball teams have entered a weekend tournament. Trophies will be given to the top 4 teams. *In how many different ways can the trophies be awarded?*

Does order matter? **YES** (1^{st} place, 2^{nd} place, …) → Permutation

$${}_nP_r = {}_{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7 = 5{,}040 \text{ ways}$$

Combinations – *A distinct group of objects that can be selected without regard to the order.* A combination occurs when the items are selected from the same group, no item is used more than once, and <u>the order does **not** matter</u>.

The Combination formula is -

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

n = the number of items in the group
r = the number of items that are being selected

Example - Your Student Council contains 24 members. They plan to form a special committee by randomly selecting 5 of their members. *In how many ways could this committee be formed?*

Does order matter? **NO** (No specific roles on committee) → Combination

$${}_nC_r = {}_{24}C_5 = \frac{24!}{(24-5)!\,5!} = \frac{24\cdot23\cdot22\cdot21\cdot20}{5\cdot4\cdot3\cdot2\cdot1} = 42{,}504 \text{ ways}$$