Graphing a Rational Function

Rational function → $f(x) = \frac{N(x)}{D(x)}$

A rational function, $f(x)$, can be graphed by following a series of steps.

Steps for Graphing -

Step 1: **Simplify $f(x)$** if possible, by factoring the numerator $N(x)$ and denominator $D(x)$. From the factorization,

A) Identify the **Domain** of the function.
B) Note any resulting "**Hole(s)**".

Step 2: Find and plot the **x-intercepts** and **y-intercept** of the function (*if they exist*).

A) x-intercept(s) → Set $N(x) = 0$ and solve.
B) y-intercept → Evaluate $f(0)$ if it exists.

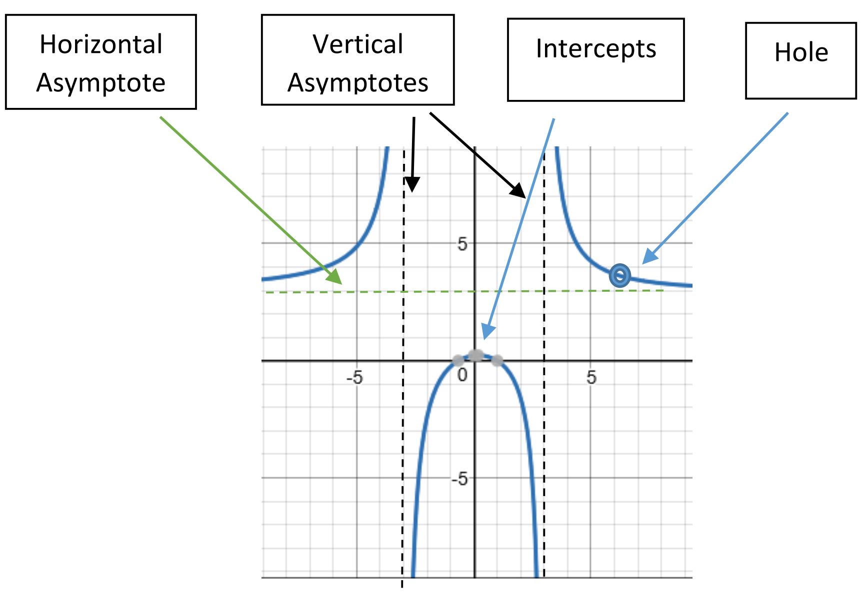
Step 3: Find and sketch any **Asymptotes** (*Horizontal, Vertical, or Slant*).

Step 4: Find and plot **additional points** if needed. Note – there should be *at least one point in between and one point beyond* each x-intercept and vertical asymptote. Constructing a **Sign Chart** and finding Origin / Y-axis **Symmetry** can also be used to aid in this step.

Step 5: Use **smooth, continuous curves** to complete the graph over each interval in the domain. In some graphs, the Horizontal Asymptote may be crossed, but **do not cross** any points of discontinuity (domain restrictions from VA's and Holes).

Step 6: Insert any identified "Hole(s)" from Step 1.

Sample Graph –



Examples – Sketch the graphs of the following rational functions.

Step 1: No Hole

a. $\boldsymbol{f(x)} = \dfrac{\mathbf{3x}}{\mathbf{x^2+2x-3}} = \dfrac{3x}{(x+3)(x-1)}$

Domain: $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$

Hole: None

Step 2: **<u>x-intercepts</u>** → $3x = 0$

$x = 0$ so $(\mathbf{0}, \mathbf{0})$

<u>y-intercept</u> → y = $\dfrac{3(0)}{(0)^2+2(0)-3} = \dfrac{0}{-3}$

$y = 0$ so $(\mathbf{0}, \mathbf{0})$

Step 3: **<u>Asymptotes</u>**

Vertical: $x + 3 = 0 \rightarrow \boldsymbol{x} = \mathbf{-3}$

$x - 1 = 0 \rightarrow \boldsymbol{x} = \mathbf{1}$

Horizontal: $n < m \quad so \quad \boldsymbol{y} = \mathbf{0}$

Step 4: **<u>Additional Points</u>**

Choose values for "*x*". Plug in to find corresponding "*y*". Plot ordered pairs (x, y).

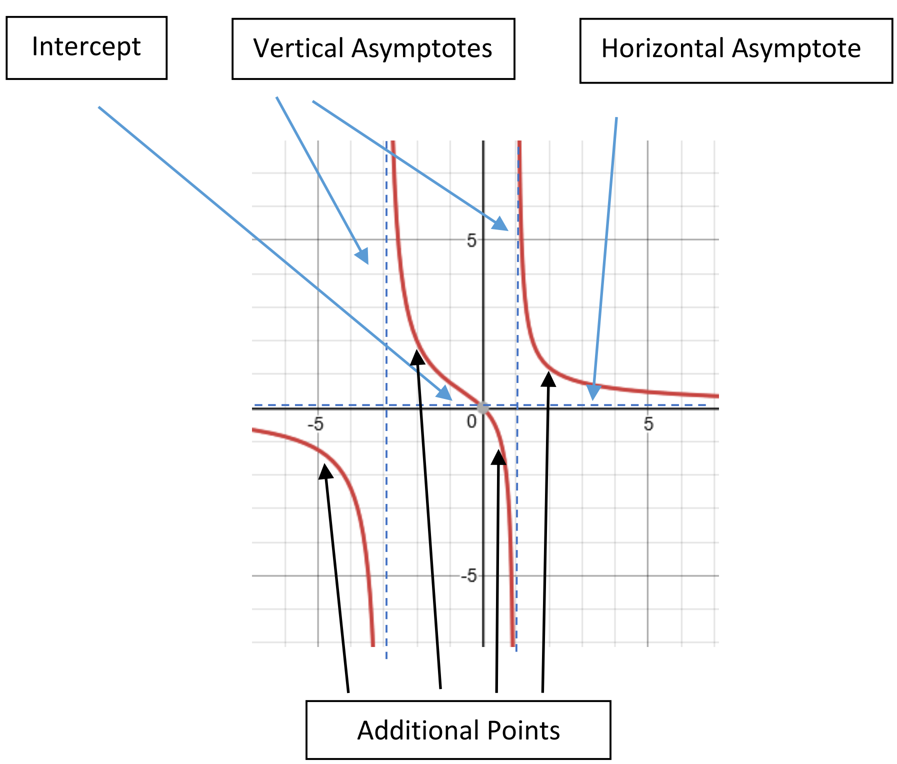
Let $x = 2 \rightarrow y = \frac{6}{5}$

$x = \frac{1}{2} \rightarrow y = -\frac{6}{7}$

$x = -2 \rightarrow y = 2$

$x = -5 \rightarrow y = -\frac{5}{6}$

Steps 5 & 6: **<u>Graph</u>**



Step 1: Hole

b. $f(x) = \frac{x^3+2x^2-x-2}{x^2+6x+8} = \frac{(x+1)(x-1)(x+2)}{(x+4)(x+2)}$ *Domain:* $(-\infty,-4) \cup (-4,-2) \cup (-2,\infty)$

Hole: $@\ x = -2$

Step 2: **x-intercepts** → $(x+1)(x-1) = 0$

$x+1 = 0,\ x-1 = 0$

$x = -1,\ 1$ so $(\pm\mathbf{1}, \mathbf{0})$

y-intercept → $y = \frac{(0+1)(0-1)}{(0+4)} = \frac{-1}{4}$

$y = -\frac{1}{4}$ so $\left(\mathbf{0}, -\frac{\mathbf{1}}{\mathbf{4}}\right)$

Step 3: **Asymptotes**

Vertical: $x+4 = 0 \rightarrow \boldsymbol{x = -4}$

Horizontal: $n > m\ by\ 1$ **None**

Slant: $\boldsymbol{y = x - 4}$ (from long division)

Step 4: **Additional Points**

Choose values for "*x*". Plug in to find corresponding "*y*". Plot ordered pairs (x, y).

Let $x = 4 \rightarrow y = \frac{15}{8}$

$x = \frac{1}{2} \rightarrow y = -\frac{1}{6}$

$x = -3 \rightarrow y = 8$

$x = -10 \rightarrow y = -\frac{33}{2}$

Find *y* coordinate of "Hole"

Let $x = -2 \rightarrow y = \frac{(-2+1)(-2-1)}{-2+4} = \frac{3}{2}$

So, Hole at $\left(-2, \frac{3}{2}\right)$

Steps 5 & 6: **Graph**