Graphing a Quadratic Function: $f(x) = ax^2 + bx + c$

Quadratic Functions are second degree polynomials *(i.e. highest power of the domain variable is 2).* Quadratics can be written in several forms - General Form, Standard Form *(also called **Vertex Form**)*, and Factored form*. The graph of a Quadratic Function is called a **Parabola**. It's general shape is curved and looks like a "U". The "U" is right side up if "a" is positive $(a > 0)$, and it is upside down if "a" is negative $(a < 0)$. The **Vertex** (h, k) is either the lowest (right side up) or the highest (upside down) point on the parabola. The **Axis of Symmetry** is a vertical line that visually cuts the parabola in half and is written as $x = h$.

General Form $(a, b, c \in \mathbb{R})$

$$f(x) = ax^2 + bx + c$$

The **y-intercept** $(0, c)$ of the graph is easily identifiable from General Form.

The **x-intercept(s)** *(if any)* can be found by factoring and/or using the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The **Vertex** (h, k), **Min/Max** value (k), and **Axis of Symmetry** $(x = h)$ can be found by completing the square or by using the vertex formula:

$$h = \frac{-b}{2a}\ ,\ k = f(h)$$

Standard *(Vertex)* Form $(a, h, k \in \mathbb{R})$

$$f(x) = a(x - h)^2 + k$$

- The **Vertex** (h, k),
- The **Min/Max** value (k) of the function, and
- The **Axis of Symmetry** $(x = h)$

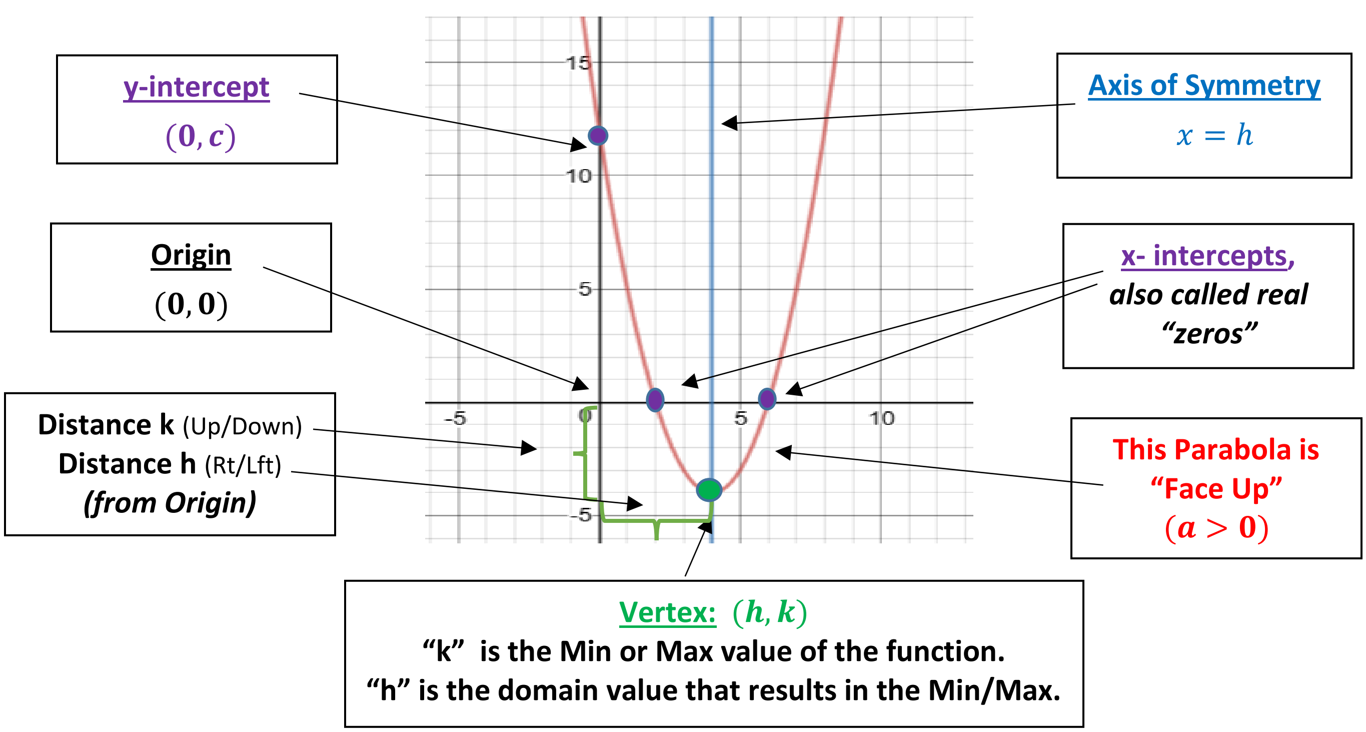
are all easily identifiable from Vertex Form.

The **x-intercept(s)** *(if any)* can be found by using the square root property.

The **y-intercept** can be found by evaluating $f(0)$.

$$f(0) = a(0 - h)^2 + k = ah^2 + k$$

Parabolic Graph of a Quadratic Function



Practice Graphing Quadratic Functions → $f(x) = ax^2 + bx + c \quad = \quad a(x-h)^2 + k$

Note: *"a" is the same number in both forms!*

Examples:

Graph the following Quadratic given in General Form: $f(x) = -3x^2 - 6x + 24$

$a \quad b \quad c$

Identify the Vertex: *(Calculate)*

$$h = \frac{-b}{2a} = \frac{-(-6)}{2(-3)} = \frac{6}{-6} = -1$$

$$k = f(h) = -3(-1)^2 - 6(-1) + 24$$
$$= 27$$

$$Vertex = (h,k) = (-1, 27)$$

Find the x-intercept(s): *(Factor or use the Quadratic Formula)*

$$f(x) = -3x^2 - 6x + 24 = 0$$
$$f(x) = -3(x^2 + 2x - 8) = 0$$
$$f(x) = -3(x+4)(x-2) = 0$$

** This is Factored Form*

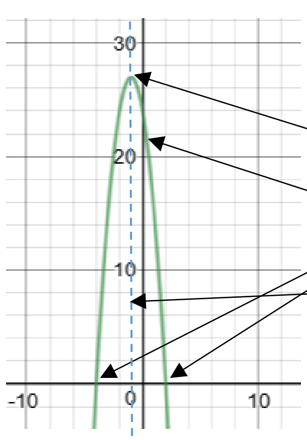
$$x + 4 = 0 \qquad x - 2 = 0$$
$$x = -4 \qquad x = 2$$

x-intercepts: $(-4, 0), \ (2, 0)$

Find the y-intercept: $(0, c) = (0, 24)$

Find the axis of symm: $x = -1$

Extra Points: Use point plotting *if needed*.



$a < 0$, So facing DOWN

Vertex (Max)

y-intercept

x-intercepts

Axis of Symmetry

Range: $(-\infty, k]$

Domain: $(-\infty, \infty)$

Graph the following Quadratic given in Standard *(Vertex)* Form: $f(x) = 3(x+1)^2 - 4$

Due to the "-" sign in Vertex Form," h" is the opposite of the number you see.

Identify the Vertex: *(from the formula)*

$$(h, k) = (-1, -4)$$

Find the x-intercept(s): *(Square root property)*

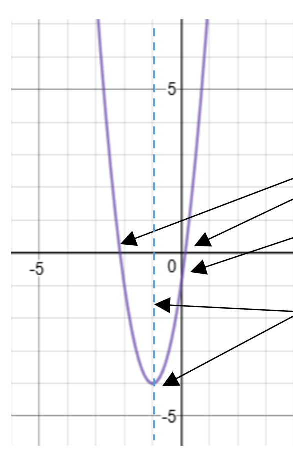
$$f(x) = 3(x+1)^2 - 4 = 0$$
$$3(x+1)^2 = 4$$
$$(x+1)^2 = \frac{4}{3}$$
$$\sqrt{(x+1)^2} = \pm\sqrt{\frac{4}{3}}$$
$$x + 1 = \pm\frac{2}{\sqrt{3}}$$
$$x = -1 \pm \frac{2\sqrt{3}}{3}$$

x-intercepts: $\left(-1 + \frac{2\sqrt{3}}{3}, 0\right), \left(-1 - \frac{2\sqrt{3}}{3}, 0\right)$

Find the y-intercept: $f(0) = 3(1)^2 - 4 = -1$
$= (0, -1)$

Find the axis of symm: $x = -1$

Extra Points: Use point plotting *if needed*.



$a > 0$, So facing UP

x-intercepts

y-intercept

Vertex (Min)

Axis of Symmetry

Range: $[k, \infty)$

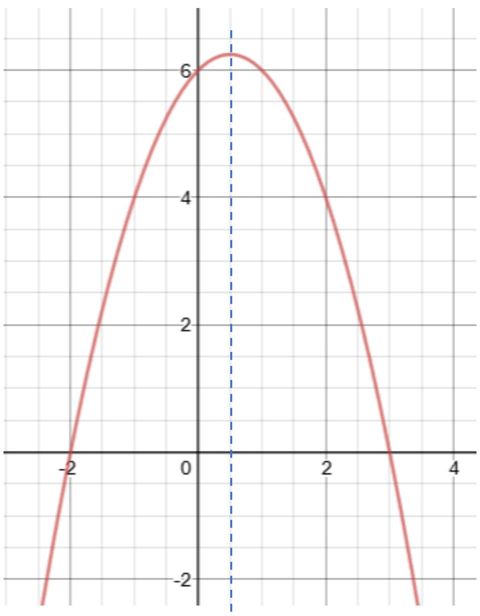
Domain: $(-\infty, \infty)$

Practice Problems: Try these on your own!

Graph the following Quadratic Functions given in *General Form*. Find the Vertex, y-intercept, and x-intercept(s) if they exist. State the Domain and the Range. Also find and show the Axis of Symmetry. State whether the parabola opens *up* or *down*.

1. $f(x) = -x^2 + x + 6$

Answer:



Vertex: $\left(\frac{1}{2}, \frac{25}{4}\right)$

y-intercept: $(0, 6)$

x-intercept(s): $(-2, 0)$, $(3, 0)$

Axis of Symmetry: $x = \frac{1}{2}$

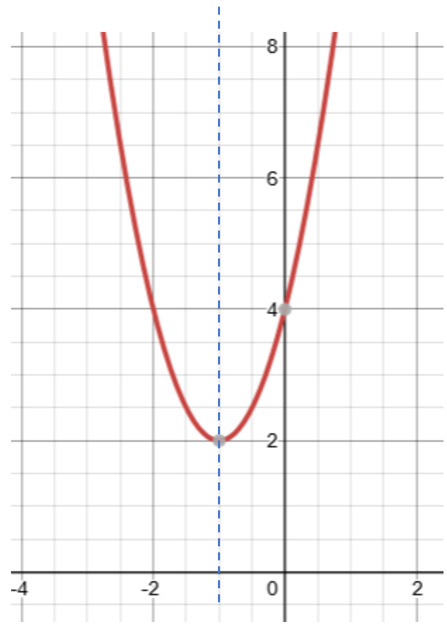
Domain: $\mathbb{R}$ or $(-\infty, \infty)$

Range: $\left(-\infty, \frac{25}{4}\right]$

Opens: Down

2. $f(x) = 2x^2 + 4x + 4$

Answer:



Vertex: $(-1, 2)$

y-intercept: $(0, 4)$

x-intercept(s): $None$

Axis of Symmetry: $x = -1$

Domain: $\mathbb{R}$ or $(-\infty, \infty)$

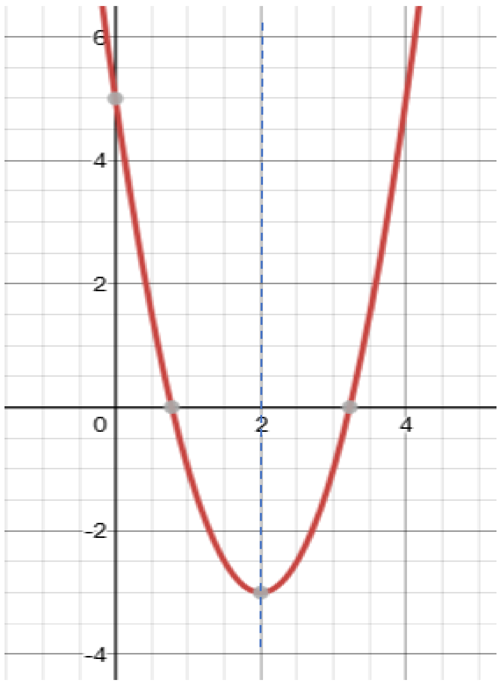
Range: $[2, \infty)$

Opens: Up

Graph the following Quadratic Functions given in *Standard (Vertex) Form*. Find the Vertex, y-intercept, and x-intercept(s) if they exist. State the Domain and the Range. Also find and show the Axis of Symmetry. State whether the parabola opens *up* or *down*.

3. $f(x) = 2(x-2)^2 - 3$

Answer:



Vertex: $(2, -3)$

y-intercept: $(0, 5)$

x-intercept(s): $\left(\frac{4+\sqrt{6}}{2}, 0\right)$, $\left(\frac{4-\sqrt{6}}{2}, 0\right)$

Axis of Symmetry: $x = 2$

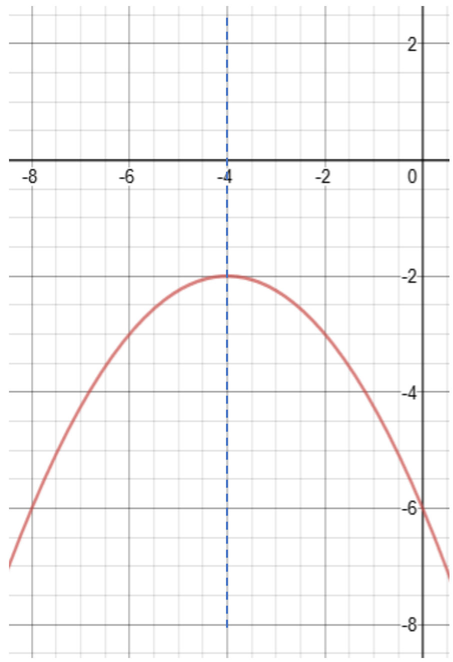
Domain: $\mathbb{R}$ or $(-\infty, \infty)$

Range: $[-3, \infty)$

Opens: Up

4. $f(x) = -\frac{1}{4}(x+4)^2 - 2$

Answer:



Vertex: $(-4, -2)$

y-intercept: $(0, -6)$

x-intercept(s): $None$

Axis of Symmetry: $x = -4$

Domain: $\mathbb{R}$ or $(-\infty, \infty)$

Range: $(-\infty, -2]$

Opens: Down