POLYNOMIAL OPERATIONS

ADDITION AND SUBTRACTION:

Adding and subtracting polynomials is the same as the procedure used in combining like terms. When *adding* polynomials, simply drop the parenthesis and combine like terms. When *subtracting* polynomials, distribute the negative first, then combine like terms.

Examples:

Addition:

$$(2x^2 + 3x - 7) + (3x^2 - 4x - 10) = 2x^2 + 3x^2 + 3x - 4x - 7 - 10 = 5x^2 - x - 17$$

Subtraction:

$$(5x^2 - 12x + 1) - (2x^2 + 3x - 7) = 5x^2 - 12x + 1 - 2x^2 - 3x + 7 = 3x^2 - 15x + 8$$

MULTIPLICATION:

1. Monomial times Monomial: To multiply a monomial times a monomial, just multiply the numbers then multiply the variables using the rules for exponents.

   Example:
   $$(-2x^2y)(5xy^7) = -2 \cdot 5x^2 \cdot x \cdot y \cdot y^7 = -10x^3y^8$$

2. Monomial times Polynomial: Simply use the distributive property to multiply a monomial times a polynomial.

   Examples:

   a. $-2x(x^2 + 3x - 8) = -2x(x^2) - 2x(3x) - 2x(-8) = -2x^3 + 6x^2 + 16x$

   b. $5x^2(-2x^4 + 3y - 6) = 5x^2(-2x^4) + 5x^2(3y) + 5x^2(-6) = -10x^6 + 15x^2y - 30x^2$

3. Binomial times a Binomial: To multiply two binomials, use the **FOIL** method (**F**irst times first, **O**utside times outside, **I**nside times inside, and **L**ast times last).

   Example:
   $$(x + 2)(x - 3) = x(x) + x(-3) + 2(x) + 2(-3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$$

   Special Products: The following formulas may be used in these special cases as a short cut to the FOIL method.

   Difference of Squares:
   $(a + b)(a - b) = a^2 - b^2$
   Example:
   $(3x + 4)(3x - 4) = 9x^2 - 16$

   Perfect Squares:
   $(a + b)^2 = a^2 + 2ab + b^2$
   Example:
   $(x + 4)^2 = x^2 + 2(x)(4) + 4^2 = x^2 + 8x + 16$

   $(a - b)^2 = a^2 - 2ab + b^2$
   Example:
   $(x - 3)^2 = x^2 - 2(x)(3) + 3^2 = x^2 - 6x + 9$

4. Polynomial times polynomial: To multiply two polynomials where at least one has more than two terms, distribute each term in the first polynomial to each term in the second.

   Examples:

   a. $(x^2+3x-4)(x^2-6x+5)$
   $$= x^2(x^2)+x^2(-6x)+x^2(5)+3x(x^2)+3x(-6x)+3x(5)-4(x^2)-4(-6x)-4(5)$$
   $$= x^4-6x^3+5x^2+3x^3-18x^2+15x-4x^2+24x-20$$
   $$= x^4-3x^3-17x^2+39x-20$$

   b. $(2x-3)(4x^2-5x+1)$
   $$= 2x(4x^2)+2x(-5x)+2x(1)-3(4x^2)-3(-5x)-3(1)$$
   $$= 8x^3-10x^2+2x-12x^2+15x-3$$
   $$= 8x^3+22x^2+17x-3$$

DIVISION:

1. Division by Monomial: Each term of the polynomial is divided by the monomial and it is simplified as individual fractions.

   Examples:

   a. $\frac{3x^2-9x+14}{3x} = \frac{3x^2}{3x} - \frac{9x}{3x} + \frac{14}{3x} = x - 3 + \frac{14}{3x}$

   b. $\frac{10y^2-25y+20}{-5y} = \frac{10y^2}{-5y} - \frac{25y}{-5y} + \frac{20}{-5y} = -2y + 5 - \frac{4}{y}$

2. Division by Binomial or Larger Polynomial:

   Use the long division format as follows:

   - Both the divisor and the dividend must be written in descending order.
   - Any missing powers should be replaced by zero.
   - All remainders are in fraction form (remainder/divisor) and are added to the quotient.

   Examples:

   a. $(x^2-2x-15) \div (x+3) = x-5$

$$\begin{array}{r} x-5 \\ x+3\overline{)\,x^2-2x-15} \\ \underline{-(x^2+3x)} \\ -5x-15 \\ \underline{-(-5x-15)} \\ 0 \end{array}$$

   b. $\frac{9x^3-x+3}{3x-2} = 3x^2+2x+1+\frac{5}{3x-2}$

$$\begin{array}{r} 3x^2+2x+1 \\ 3x-2\overline{)\,9x^3+0x^2-x+3} \\ \underline{-(9x^3-6x^2)} \\ 6x^2-x \\ \underline{-(6x^2-4x)} \\ 3x+3 \\ \underline{-(3x-2)} \\ 5 \end{array}$$

POLYNOMIAL OPERATIONS PRACTICE

Add the following polynomials (Write answers in descending order):

1. $(7j^3 - 2) + (5j^3 - j - 3)$
2. $(8a^5 - 4) + (3a^5 + a - 2)$
3. $(6m^5 + 1) + (2m^5 + 9m - 1)$
4. $(3m^5 + 1) + (9m^5 + 3m - 2)$
5. $(-5x^2 - x + 4) + (-3x^2 - 5x + 2)$
6. $(-4x + 4x^3 + 7) + (3x^3 - 9 - 3x)$
7. $(3x^2 - 2x + 1) + (-x^2 + 3x + 1)$

Subtract the following polynomials (Write answers in descending order):

8. $(-x^2 + x - 4) - (3x^2 - 8x - 2)$
9. $(8x^2 - 3x) - (5x - 5 - 8x^2)$
10. $(-x^2 - 5x - 3) - (-7x^2 - 8x - 8)$
11. $(-2x^3 + x) - (7x - 3 - 7x^3)$
12. $(3x^3 + 3x^2 + 9) - (5x^3 - 7x^2 + 6x - 9)$
13. $(5x^3 + 5x^2 + 5) - (6x^3 - 6x^2 + 8x - 5)$
14. $(5x^3 + 3x^2 + 5) - (7x^3 - 9x^2 + 8x - 5)$

Multiply the following polynomials:

15. $(8x^3y^2)(-3x^2y^3)$
16. $(-9x^3y)(-8x^2y^3)$
17. $j^2(k^5j^3)$
18. $a^4(b^4a^6)$
19. $2x^3(9x^2 + 5y)$
20. $5x^3(2x + 4y)$
21. $5m^2(3m^3 + 5m^2 - 4m + 6)$
22. $-4x^2y(x^2 + 7xy - 6y^3)$
23. $(x + 6)(x + 2)$
24. $(x - 6)(x + 9)$
25. $(4x - 3)(3x - 5)$
26. $(x - 8)(x - 7)$
27. $(6a + 1)(5a + 2)$
28. $(5x + 4y)(2x + 5y)$
29. $(2x + y)(4x - 9y)$
30. $(6r - 5)(6r + 1)$
31. $(6c + 7)(6c - 7)$
32. $(3x + 5y)^2$
33. $(x - 2)(x^2 - x + 3)$
34. $(2x - 5)(5x^2 + 4x + 7)$

Divide the following polynomials:

35. $\frac{9x-6}{3}$
36. $\frac{4x-7}{2}$
37. $\frac{x^2-3x+5}{x}$
38. $\frac{5x^2-25x+2}{-5x}$
39. $\frac{4x^{10}-5x^9-20x^4}{4x^2}$
40. $(-x^6 + x^5 + 7x^2 - 9) \div x^4$
41. $(x^2 + 2x + 6) \div x$
42. $(3x^2 - 15x + 5) \div (-3x)$
43. $(2x^{11} - 5x^7 - 10x^6) \div 2x^3$
44. $(-2x^6 + 5x^5 + 9x^2 + 2) \div x^4$
45. $\frac{f^3+64}{f+4}$
46. $\frac{4p-2+3p^2}{p-1}$
47. $\frac{3m-4+2m^2}{m+5}$
48. $\frac{j^3-64}{j-4}$
49. $\frac{-5p+4p^2+4}{p-2}$
50. $(4p + 3p^2 - 1) \div (p + 4)$
51. $(20x^2 - 13x + 2) \div (5x - 2)$
52. $(12x^2 - 6x^3 - 3 - 9x) \div (3x - 3)$
53. $(8x^2 - 2x - 3) \div (2x + 1)$
54. $(-3x^2 + 6x^3 - 4 - x) \div (2x + 1)$

POLYNOMIAL OPERATIONS PRACTICE ANSWERS

1. $12j^3 - j - 5$
2. $11a^5 + a - 6$
3. $8m^5 + 9m$
4. $12m^5 + 3m - 1$
5. $-8x^2 - 6x + 6$
6. $7x^3 - 7x - 2$
7. $2x^2 + x + 2$
8. $-4x^2 + 9x - 2$
9. $16x^2 - 8x + 5$
10. $6x^2 + 3x + 5$
11. $5x^3 - 6x + 3$
12. $-2x^3 + 10x^2 - 6x + 18$
13. $-x^3 + 11x^2 - 8x + 10$
14. $-2x^3 + 12x^2 - 8x + 10$
15. $-24x^5y^5$
16. $72x^5y^4$
17. j^5k^5
18. $a^{10}b^4$
19. $18x^5 + 10x^3y$
20. $10x^4 + 20x^3y$
21. $15m^5 + 25m^4 - 20m^3 + 30m^2$
22. $-4x^4y - 28x^3y^2 + 24x^2y^4$
23. $x^2 + 8x + 12$
24. $x^2 + 3x - 54$
25. $12x^2 - 29x + 15$
26. $x^2 - 15x + 56$
27. $30a^2 + 17a + 2$
28. $10x^2 + 33xy + 20y^2$
29. $8x^2 - 14xy - 9y^2$
30. $36r^2 - 24r - 5$
31. $36c^2 - 49$
32. $9x^2 + 30xy + 25y^2$
33. $x^3 - 3x^2 + 5x - 6$
34. $10x^3 - 17x^2 - 6x - 35$
35. $3x - 2$
36. $2x - \frac{7}{2}$
37. $x - 3 + \frac{5}{x}$
38. $-x + 5 - \frac{2}{5x}$
39. $x^8 - \frac{5x^7}{4} - 5x^2$
40. $-x^2 + x + \frac{7}{x^2} - \frac{9}{x^4}$
41. $x + 2 + \frac{6}{x}$
42. $-x + 5 - \frac{5}{3x}$
43. $x^8 - \frac{5x^4}{2} - 5x^3$
44. $-2x^2 + 5x + \frac{9}{x^2} + \frac{2}{x^4}$
45. $f^2 - 4f + 16$
46. $3p - 7 + \frac{5}{p-1}$
47. $2m - 7 + \frac{31}{m+5}$
48. $j^2 + 4j + 16$
49. $4p + 3 + \frac{10}{p-2}$
50. $3p - 8 + \frac{31}{p+4}$
51. $4x - 1$
52. $-2x^2 + 2x - 1 - \frac{2}{x-1}$
53. $4x - 3$
54. $3x^2 - 3x + 1 - \frac{5}{2x+1}$