MATH 180 FINAL REVIEW

For Problems # 1 – 8, find the limit if it exists:

1. $\lim_{x \to 3} \frac{\sqrt{x+6}}{x+2}$

   a. $\frac{1}{2}$ b. $-\sqrt{3}$ c. $\frac{9}{5}$ d. $\frac{3}{5}$

2. $\lim_{x \to 2} \frac{x^2+2x-8}{x^2-x-2}$

   a. $Does\ not\ exist$ b. $\frac{1}{2}$ c. 2 d. ∞

3. $\lim_{x \to 8^+} \frac{|x-8|}{x-8}$

   a. -1 b. 1 c. $Does\ not\ exist$ d. ∞

4. $\lim_{x \to 4^-} \frac{\sqrt{x}-2}{x-4}$

   a. $\frac{1}{4}$ b. 1 c. $\frac{1}{6}$ d. $Does\ not\ exist$

5. $\lim_{x \to 0} \frac{Sin\ 2x}{Sin\ x}$

   a. 1 b. 2 c. 0 d. $Does\ not\ exist$

6. $\lim_{\theta \to 0} \frac{Cos\theta Tan\theta}{\theta}$

   a. 0 b. ∞ c. 1 d. $Does\ not\ exist$

7. $\lim_{x \to -\infty} \frac{x}{\sqrt{x^2+1}}$

   a. 1 b. ∞ c. $-\infty$ d. -1

8. $\lim\limits_{n \to \infty} \frac{n^3-4}{n^2+1}$

a. -4 b. 1 c. -1 d. $Does\ not\ exist$

9. Find any *x values* where $f(x)$ is **not** continuous. $f(x) = \frac{x+2}{x^2-2x-8}$

a. 4 b. -4, 2 c. -2, 4 d. -4, -2, 2

10. Find an equation of the line that is tangent to the graph of the function at the given point.

$f(x) = \sqrt{x-1}$ $(5, 2)$

a. $x - 4y = 3$ b. $4x - y = -3$ c. $x - 4y = -3$ d. $4x - y = 18$

For Problems # 11 – 14, find the derivative $(f'(x))$ of the function and evaluate if requested.

11. $f(x) = x\,(2x-5)^3$

a. $(2x-5)^2(8x-5)$ b. $3x(2x-5)^2$

c. $6x(2x-5)^2$ d. $5(x-1)(2x-5)^2$

12. $f(x) = \frac{Cos\ x}{Csc\ x}$

a. $Cos\ 2x$ b. $Sin\ 2x$ c. 1 d. $\cos^{2x} + \sin^2 x$

13. $f(x) = \tan^2 x$ Evaluate at the point $\left(\frac{\pi}{4}, 1\right)$.

a. 2 b. 1 c. 4 d. $\frac{1}{2}$

14. $f(x) = \frac{x}{\sqrt{x^2+1}}$ Evaluate at the point $\left(1, \frac{\sqrt{2}}{2}\right)$

a. $\frac{\sqrt{2}}{2}$ b. $\frac{\sqrt{2}}{4}$ c. $2\sqrt{2}$ d. $\frac{1}{2}$

For Problems # 15 – 16, use implicit differentiation to find $\frac{dy}{dx}$.

15. $\sqrt{x\,y} = x^2y + 1$

a. $\frac{-2}{\sqrt{xy}-4xy}$ b. $\frac{4x\ \sqrt{xy}-y}{x-2x^2\sqrt{xy}}$ c. $\frac{4xy}{x-2x^2}$ d. $\frac{4xy\sqrt{xy}-1}{1-2x^2\sqrt{xy}}$

16. $4xy + \ln(x^2y) = 7$

a. $\frac{-2y(2xy-1)}{x(4xy+1)}$ b. $\frac{-4xy^2-2y}{4x^2y+x}$ c. $\frac{xy}{2+4xy}$ d. $\frac{-6xy^2}{4x^2y+1}$

For Problems # 17 – 19, differentiate.

17. $f(x) = ln\sqrt{x^2 - 4}$

a. $\frac{1}{x^2-4}$ b. $\frac{x}{x^2-4}$ c. $\frac{2x}{\sqrt{x^2-4}}$ d. $\frac{1}{2\,(x^2-4)}$

18. $f(x) = x\,e^{2x}$

a. $e^{2x}(x+1)$ b. $e^{2x}(x+2)$ c. $2xe^{2x}$ d. $e^{2x}(2x+1)$

19. $f(x) = 5^{-4x}$

a. $-20(5^{-4x})$ b. $\frac{-4\,(5^{-4x})}{ln5}$ c. $\frac{-4ln5}{625^x}$ d. $-4(ln5)5^{4x}$

20. An isosceles triangle has two sides of equal length s and an included angle θ. If the angle θ is increasing at a rate of $\frac{1}{2}$ $radian\ per\ minute$, find the rate of change of the *Area* of the triangle when $\theta = \frac{\pi}{6}$. Use the following formula for the Area of the triangle: $A = \frac{s^2}{2}sin\theta$.

a. $\frac{s^2}{8}$ b. $\frac{\sqrt{3}\,s^2}{8}$ c. $\frac{\sqrt{3}\,s^2}{4}$ d. $\frac{\sqrt{3}\,s}{8}$

21. A spherical balloon is inflated with helium at a rate of 800 m^3/min . How fast is the *radius* of the balloon changing at the instant the radius is 60 cm?

a. $\frac{10}{3\pi}\ cm/min$ b. $\frac{\pi}{18}cm/min$ c. $\frac{10}{9\pi}cm/min$ d. $\frac{1}{18\pi}\ cm/min$

In Problems # 22 – 23, find the indicated absolute extrema on the given interval.

22. $y = 2x^3 - 6x \quad [0, 3]$ Absolute *maximum*

a. $(3, 36)$ b. $(1, -4)$ c. $(3, 48)$ d. $(-1, 4)$

23. $y = 3\ cosx \quad [0, 2\pi]$ Absolute *minimum*

a. $(2\pi, 3)$ b. $(1, -3)$ c. $(0, 3)\ and\ (2\pi, 3)$ d. $(\pi, -3)$

In Problems # 24 – 25, find the intervals where the function is increasing or decreasing as indicated.

24. $f(x) = -3x^2 - 4x - 2 \quad (increasing)$

a. $\left(-\infty, \frac{2}{3}\right]$ b. $\left[\frac{3}{2}, \infty\right)$ c. $\left(-\infty, -\frac{2}{3}\right)$ d. $\left(-\frac{2}{3}, \infty\right)$

25. $f(x) = \frac{x}{x-5} \quad (decreasing)$

a. $(-\infty, \infty)$ b. $(-\infty, 5)$ c. $(-\infty, 5), (5, \infty)$ d. $(5, \infty)$

In Problems # 26 – 27, find the intervals where the function is concave up or concave down as indicated.

26. $f(x) = -3x^4 - x + 4 \quad (Concave\ Down)$

a. $(-\infty, 0)$ b. $(-\infty, \infty)$ c. $(0, \infty)$ d. $(-\infty, 0), (0, \infty)$

27. $f(x) = x + 2cosx \quad [0, 2\pi] \quad (Concave\ Up)$

a. $\left(\frac{7\pi}{6}, \frac{11\pi}{6}\right)$ b. $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ c. $(0, \pi)$ d. $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$

28. Find the Vertical Asymptote(s), if any, of the following function. $g(x) = \frac{6x}{36-x^2}$

a. $x = -6, x = 6$ b. $y = 0$ c. $x = 0$ d. $y = -6,\ y = 6$

29. Which limit should be used to find the Horizontal Asymptote(s), if any, of the following function?

$h(x) = \frac{5x^2-2}{x^2}$

a. $\lim_{x \to \infty} h(x)$ b. $\lim_{x \to -\infty} h(x)$ c. Neither a or b d. Both a and b

In Problems # 30 – 33, evaluate the indefinite integrals.

30. $\int(\sqrt[4]{x^3}+1)\,dx$

a. $\frac{4}{7}x^{\frac{7}{4}}+x+C$ b. $\frac{3}{7}x^{\frac{7}{3}}+x+C$ c. $\frac{7}{4}x^{\frac{7}{4}}+x+C$ d. $\frac{3}{4}x^{\frac{-1}{4}}+C$

31. $\int\frac{6x^2}{(4x^3-9)^3}\,dx$

a. $\frac{(4x^3-9)^4}{2}+C$ b. $\frac{-1}{4(4x^3-9)^2}+C$ c. $\frac{-1}{(4x^3-9)^2}+C$ d. $\frac{4x}{(4x^3-9)^2}+C$

32. $\int\frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx$

a. $ln|e^x+e^{-x}|+C$ b. $ln|e^x-e^{-x}|+C$ c. $\frac{(e^x+e^{-x})^2}{2}+C$ d. $e^x+e^{-x}+C$

33. $\int\frac{2x-5}{x^2+2x+2}\,dx$

a. $ln|x^2+2x+2|-3x+C$ b. $2\arctan(x+1)+C$

c. $\frac{1}{x+2}+C$ d. $ln|x^2+2x+2|-7\arctan(x+1)+C$

In Problems #34 – 38, evaluate the definite integrals.

34. $\int_{-1}^{1}\frac{x^2+2x+1}{x^4}\,dx$

a. -2 b. -4 c. $-\frac{8}{3}$ d. 0

35. $\int_0^5|2x-5|\,dx$

a. 25 b. $\frac{25}{2}$ c. -25 d. $-\frac{25}{2}$

36. $\int_0^\pi(2+cosx)dx$

a. 2π b. $2\pi-1$ c. $2\pi+1$ d. $\frac{\pi}{2}$

37. $\int_{ln}^{ln} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\, dx$

a. $\frac{3}{16}$ b. $\arcsin(4) - arcsin(2)$ c. $\frac{\pi}{3} - \arccos\left(\frac{1}{4}\right)$ d. $\frac{\pi}{6} - \arcsin\left(\frac{1}{4}\right)$

38. $\int_0^{\frac{\pi}{2}} \frac{cosx}{1+\sin^2 x}$ dx

a. $\frac{\pi}{4}$ b. 0 c. $-\frac{\pi}{4}$ d. $ln2$

Additional Problems:

1. For what value of *c* is $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2cx, & x \geq 3 \end{cases}$ continuous at every *x*?
2. If $f(x) = \frac{x^2}{x-3}$, find the following:
   a. The open intervals where *f* is increasing or decreasing.
   b. The open intervals where *f* is concave up or concave down.
   c. Relative extrema if they exist.
   d. Points of inflection if they exist.
   e. Vertical, horizontal, and slant asymptotes.

3. Prove the statement using the precise definition of a limit (i.e. use a δ, ε proof):
   $\lim_{x \to 4}(5x - 7) = 13$

4. A printer needs to make a poster that will have a total area of 200 square inches and will have 1 inch margins on the sides, a 2 inch margin on the top and a 1.5 inch margin on the bottom as shown below. What dimensions will give the largest printed area? Round to the nearest tenth of an inch.

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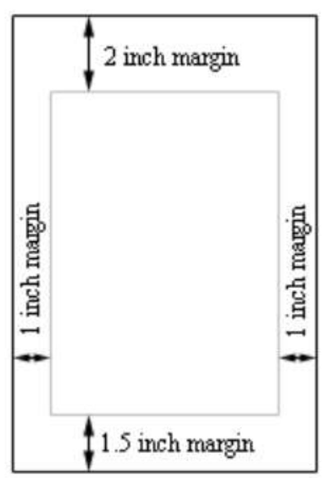
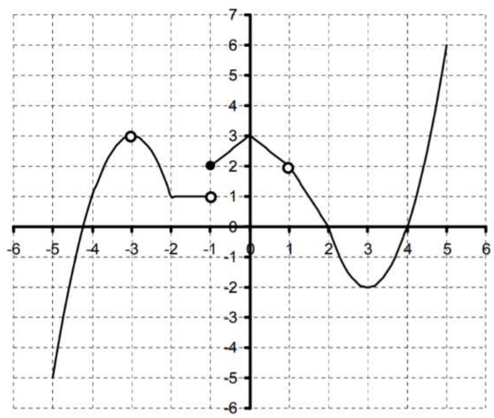


5. A sphere was measured and its radius was found to be 45 inches with a possible error of 0.01 inches. What is the maximum possible error in the volume?
6. Find the derivative of the function $f(x) = 2x^2 - 3x + 7$ using the limit process.

7. Consider the following graph of the function f. Find each limit, if it exists. If a limit does not exist, state that fact.



a. $\lim_{x \to -3} f(x) =$ ________

b. $\lim_{x \to 0} f(x) =$ ________

c. $\lim_{x \to -2} f(x) =$ ________

d. $\lim_{x \to -1^-} f(x) =$ ________

e. $\lim_{x \to -1^+} f(x) =$ ________

f. $\lim_{x \to -1} f(x) =$ ________

8. Evaluate $\lim_{x \to 0} \frac{|x|}{x}$ numerically by filling in the table below:

x	-3	-2	-1	0	1	2	3
$\frac{\lvert x \rvert}{x}$							

$\lim_{x \to 0} \frac{|x|}{x} =$ ___________

9. Use the Fundamental Theorem of Calculus to find the area of the region bounded by the graphs of the equations $y = 3^{\cos x} \sin x$, $y = 0$, $x = 0$, $x = \pi$.

10. Find the equation of the tangent line to the graph of the function $y = \sinh(1 - x^2)$ at the point (1, 0).

ANSWER KEY

1.	d	15.	b	29.	d
2.	c	16.	b	30.	a
3.	b	17.	b	31.	b
4.	a	18.	d	32.	a
5.	b	19.	c	33.	d
6.	c	20.	b	34.	c
7.	d	21.	d	35.	b
8.	d	22.	a	36.	a
9.	c	23.	d	37.	d
10.	c	24.	c	38.	a
11.	a	25.	c		
12.	a	26.	b		
13.	c	27.	b		
14.	b	28.	a		

Additional Problems:

1. c = 4/3
2. a. Increasing: $(-\infty, 0) \cup (6, \infty)$ Decreasing: $(0,3) \cup (3,6)$
   b. Concave up: $(3, \infty)$ Concave down: $(-\infty, 3)$
   c. Relative maximum: (0,0) Relative minimum: (6, 12)
   d. None
   e. Vertical asymptote: x=3 Horizontal asymptote: None Slant asymptote: y=x+3
3. Proof: Given ε>0. Choose δ= ε/5.
   If 0 < $|x-4| < \delta$, then $|5x-7-13| = |5x-20| = 5|x-4| < 5\delta = 5 \cdot \frac{\varepsilon}{5} = \varepsilon$.
   Thus, by definition of a limit, $\lim_{x \to 4}(5x-7) = 13$.
4. Length: 18.7 inches Width: 10.7 inches
5. 81π $in.^3$

6. Proof:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 - 3(x+\Delta x) + 7 - (2x^2 - 3x + 7)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x^2 + 4x\Delta x + 2(\Delta x)^2 - 3x - 3\Delta x + 7 - 2x^2 + 3x - 7}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4x\Delta x + 2(\Delta x)^2 - 3\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 4x + 2\Delta x - 3$$

$$= 4x - 3$$

7. a. 3 b. 3 c. 1 d. 1 e. 2 f. DNE

8.

x	-3	-2	-1	0	1	2	3
$\frac{\lvert x \rvert}{x}$	-1	-1	-1		1	1	1

$\lim_{x \to 0} \frac{|x|}{x}$ = DNE

9. $\frac{8}{3ln3}$

10. y = 2x-2