Math 150 – Pre-Calculus
Final Exam Review (Fall 2024)

Part I. Multiple Choice: Choose the best possible answer.

1. Find the domain of the function: $f(x) = \sqrt{2x+10} - 4$

   a. $(-\infty, -5] \cup [-4, \infty)$ b. $(-5, -\infty)$ c. $[-5, \infty)$ d. $[-4., \infty)$

2. Find the distance between the points: $P = (-7, 3)$ & $Q = (4, 5)$

   a. $\sqrt{13}$ b. $5\sqrt{5}$ c. $\sqrt{17}$ d. $\sqrt{17}i$

3. Which of the following equations represent *y <u>as a function</u> of x*?

   a. $\frac{x^2}{4} - \frac{y^2}{9} = 1$ b. $y = 3x^2 + 9$ c. $|y| = x - 10$ d. $x^2 + y^2 = 16$

4. Find the average rate of change of the function $f(x) = x^2 - 2x + 8$ from $x_1 = 2$ to $x_2 = 5$.

   a. 5 b. 3 c. -5 d. 11

5. Is $f(x) = x^4 - 2x^2 + 3$ even, odd, or neither? Does it have any symmetry?

   a. Odd with origin symmetry b. Even with x-axis symmetry

   c. Neither with no symmetry d. Even with y-axis symmetry

6. Find the inverse function $f^{-1}(x)$ of $f(x) = \frac{x+1}{x-2}$

   a. $f^{-1}(x) = \frac{2x-1}{x+1}$ b. $f^{-1}(x) = \frac{-2x+1}{x-1}$

   c. $f^{-1}(x) = \frac{x-2}{x+1}$ d. $f^{-1}(x) = \frac{2x+1}{x-1}$

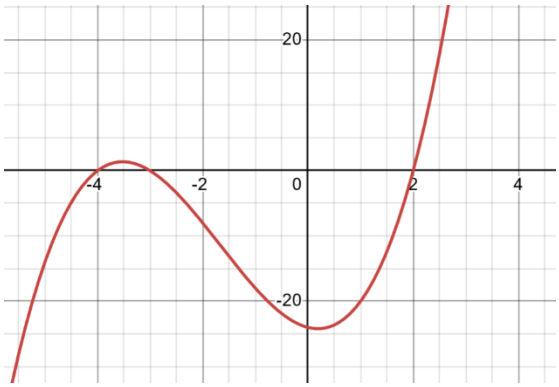
7. Solve for x: $2x^2 - 5x + 3 \geq 0$

   a. $\left(-\infty, 1\right] \cup \left[\frac{3}{2}, \infty\right)$ b. $\left[1, \frac{3}{2}\right]$ c. $(-\infty, 1) \cup \left(\frac{3}{2}, \infty\right)$ d. $\left(1, \frac{3}{2}\right)$

8. Given the following piecewise function, find $f(1)$: $f(x) = \begin{cases} x^3 - x & x \leq 1 \\ x + 4 & x > 1 \end{cases}$

a. 3 b. 5 c. 2 d. 0

9. Find the equation of the polynomial graphed below.



a. $y = (x-2)^2(x+3)(x+4)$ b. $y = (x+2)^2(x-3)(x-4)$

c. $y = (x-2)(x+3)(x+4)$ d. $y = (x+2)(x-3)(x-4)$

10. Find the equation of the secant line for $f(x) = x^2 + 3x + 1$ on the interval $[2\,,7]$.

a. $y = 2x - 13$ b. $y = 12x - 13$

c. $y = 12x - 130$ d. $y = 12x - 17$

Given $\boldsymbol{u} = \langle -2\,,5\rangle$ and $\boldsymbol{v} = \langle -1\,,8\rangle$, answer questions 11 and 12.

11. Find the dot product: $\boldsymbol{u} \cdot \boldsymbol{v}$

a. -38 b. 41 c. 42 d. 60

12. Find the angle between the vectors to the nearest tenth of a degree:

a. 138.4° b. 14.7° c. 36.2° d. 14.67°

13. Expand the expression by using the properties of logarithms:

$$log_2\left(\frac{4m\sqrt{n}}{p^2}\right)$$

a. $log_2 4 + log_2 m + \frac{1}{2} log_2 n - 2log_2 p$

b. $log_2 4m + \frac{1}{2} log_2 n - 2log_2 p$

c. $log_2 4 + log_2 m + \frac{1}{2} log_2 n + 2log_2 p$

d. $2 + log_2 m + \frac{1}{2} log_2 n - 2log_2 p$

14. Given $\cos u = -\frac{2}{7}$ and $\frac{\pi}{2} < u < \pi$, find $\cos\frac{u}{2}$ and $\sin 2u$.

a. $\cos\frac{u}{2} = -\frac{\sqrt{70}}{14}$ $\quad \sin 2u = \frac{6\sqrt{5}}{7}$

b. $\cos\frac{u}{2} = \frac{\sqrt{70}}{14}$ $\quad \sin 2u = -\frac{6\sqrt{5}}{7}$

c. $\cos\frac{u}{2} = \frac{3\sqrt{14}}{14}$ $\quad \sin 2u = -\frac{12\sqrt{5}}{49}$

d. $\cos\frac{u}{2} = \frac{\sqrt{70}}{14}$ $\quad \sin 2u = -\frac{12\sqrt{5}}{49}$

For problems 16 and 17, let $\sin A = -\frac{7}{25}$ with A in Quadrant III and $\cos B = -\frac{4}{5}$ with B in Quadrant III.

15. Find $\sin(A + B)$

a. $-\frac{4}{5}$ b. $\frac{3}{5}$ c. $\frac{4}{5}$ d. $-\frac{3}{5}$

16. Find $\tan(A - B)$

a. $\frac{100}{117}$ b. $-\frac{44}{75}$ c. $\frac{44}{75}$ d. $-\frac{44}{117}$

17. Simplify the trigonometric expression: $\frac{\sec\theta - 1}{1 - \cos\theta}$

a. $\sec\theta$ b. $\cos\theta$ c. $\frac{\sec\theta + \cos\theta}{\sin^2\theta}$ d. -1

18. Simplify the trigonometric expression: $\frac{1}{\cos x+1}+\frac{1}{\cos x-1}$

a. $\sec x$ b. $-2\csc x\cot x$ c. $-2\csc^2 x$ d. $\frac{2}{\cos^2 x-1}$

19. Evaluate $\tan^{-1}(-1)$

a. $\frac{3\pi}{4}, \frac{7\pi}{4}$ b. $\frac{7\pi}{4}$ c. $-\frac{\pi}{4}$ d. $\frac{3\pi}{4}$

20. Evaluate $sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$

a. $\frac{\sqrt{3}}{2}$ b. $-\frac{\sqrt{3}}{2}$ c. $\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$ d. $\frac{\sqrt{3}\,\pi}{2}$

21. In triangle ABC, if $a = 3.7\ cm, c = 6.4\ cm, and\ B = 23°,$ find $b.$

a. 4.1 cm b. 3.3 cm c. 5.7 cm d. 11.1 cm

Part II. Short Answer Section: Show your work.

1. Find the difference quotient: $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$ for $f(x) = 5x - x^2$.

2. The graph of $f(x) = \sqrt{x}$ is shifted up 3 units, right 4 units, reflected about the y-axis, and compressed vertically by a factor of 3. Name the resulting function g, then write an equation for $g(x)$ in terms of f.

3. Describe the transformations that would produce the graph of g from the graph of f. Be specific and detailed. If more than one transformation is needed, specify the order in which the transformations should be applied.

   a. $g(x) = f(2x) + 7$

   b. $g(x) = -f(x - 3) - 5$

4. Find the compositions $(f \circ g)(x)$ and $(g \circ f)(x)$ using the following functions: $f(x) = \sqrt[3]{x - 5}$ and $g(x) = x^3 + 1$. Are $f(x)$ and $g(x)$ inverse functions of each other? Explain why or why not.

5. Given the function. $f(x) = x^3 + 2x^2 + 4x + 8$.

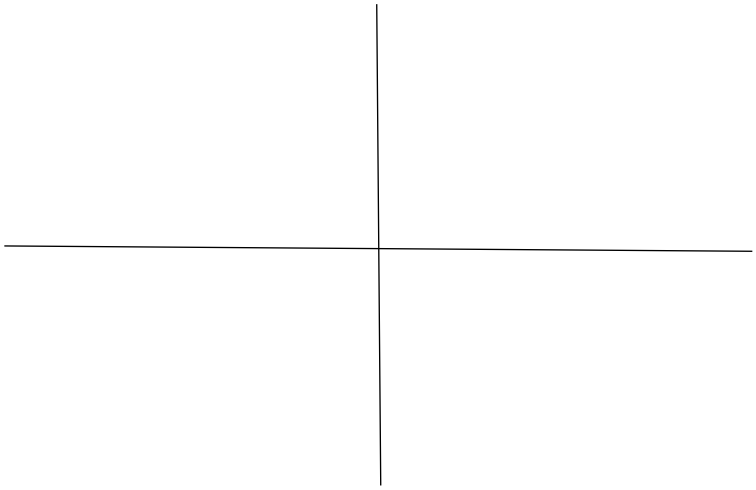
a. Factor the polynomial over the real numbers as the product of linear factors or irreducible quadratic factors.

b. State all zeros (real and imaginary) and their associated multiplicities.

c. State the end behavior.

d. Find the y-intercept.

e. Using the information in parts a – d above as a guide, sketch $f(x)$.



6. Find all asymptotes that exist (vertical, horizontal and/or slant) for the following function: $f(x) = \frac{-4x^2+1}{x^2+x-2}$. If they do not exist, explain why. State the domain for the function.

Vertical Horizontal Slant

Domain:

7. Solve the following exponential and logarithmic equations. Leave answers in exact form:

a. $8^x = 32^{x-1}$

b. $5^x + 8 = 26$

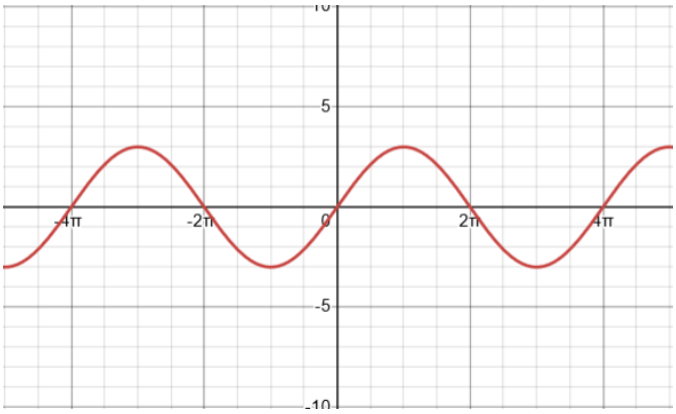
c. $log_2 x + log_2(x+2) = log_2(x+6)$

d. $log(8x) - log(x+1) = 2$

8. Given $\sin\theta = -\frac{12}{15}$ and θ terminates in Quadrant III, find the five remaining trigonometric functions of θ.

9. Solve the following non-linear system: $\begin{cases} x - 2y = -6 \\ x^2 - y = 0 \end{cases}$ by substitution.

10. Write a trigonometric function matching the graph below using a **sine** function:



11. Given the following trigonometric function: $y = 2 + 2\sec(x - \frac{\pi}{4})$

Find the period, amplitude, horizontal translation, and vertical translation.

Amplitude:

Period:

Horizontal Translation:

Vertical Translation:

12. Solve the trigonometric equations:

a. $\csc^2 x + 3\csc x - 4 = 0$ over $[0, 2\pi)$

b. $2\sin^2 x + 5\cos x - 4 = 0$

c. $2\sin 2x + \sqrt{3} = 0$ over $[0, 2\pi)$

d. $\sec 4x - 2 = 0$

Answer Key

Part 1: Multiple Choice Section

1.	C	10.	B	19.	A
2.	B	11.	B	20.	A
3.	B	12.	B	21.	B
4.	A	13.	D		
5.	D	14.	D		
6.	D	15.	C		
7.	A	16.	D		
8.	D	17.	C		
9.	C	18.	B		

Part 2: Short Answer Section

1. $5 - 2x - h$

2. $g(x) = \frac{1}{3}\sqrt{-x+4} + 3$ & $g(x) = \frac{1}{3}f(-x+4) + 3$

3a. 1. Horizontal compression by a factor of 2

2. Horizontal shift 7 units up

3b. 1. Horizontal shift right by 3 units

2. Reflect about the x-axis

3. Vertical shift down by 5 units

4. $(f \circ g)(x) = \sqrt[3]{x^3 - 4}$, $(g \circ f)(x) = x - 4$,

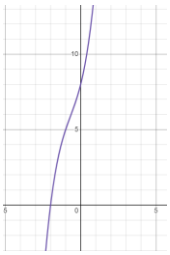
No, they are NOT inverses because $(f \circ g)(x) \neq (g \circ f)(x) \neq x$

5a. $f(x) = (x+2)(x^2+4)$

5b. Zeros: $x = -2\ (mult\ 1), x = 2i\ (mult\ 1), x = -2i\ (mult\ 1)$

5c. Down left, Up right

5d. $(0, 8)$

5e. 

6. VA: $x = -2,\ x = 1$

HA: $y = -4$

Slant: No slant asymptote. Power of numerator is not greater than power of denominator by 1.

Domain: $(-\infty, -2) \cup (-2,1) \cup (1, \infty)$

7a. $x = \frac{5}{2}$,

7b. $x = \frac{\ln 18}{\ln 5}\ OR = \frac{\log 18}{\log 5}$ OR $= log_5 18$

7c. $x = 2$

7d. No Solution

8. $\csc\theta = -\frac{15}{12}\ \cos\theta = -\frac{9}{15}$ $\sec\theta = \frac{-15}{9}\ \tan\theta = \frac{12}{9}\ \cot\theta = \frac{9}{12}$

9. $(2, 4), (-\frac{3}{2}, \frac{9}{4})$

10. $f(x) = 3\sin\left(\frac{x}{2}\right)$

11. *Amplitude:* none
Period: 2π
Horizontal Translation: $\frac{\pi}{4}$ to the right
Vertical Translation: 2 up

12a. $x = \frac{\pi}{2},\ \ x = \arcsin\left(\frac{1}{4}\right),\ \ x = \arcsin\left(-\frac{1}{4}\right)$

12b. $x = \frac{\pi}{3} + 2n\pi,\ x = \frac{5\pi}{3} + 2n\pi$

12c. $\frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

12d. $x = \frac{\pi}{12} + \frac{n\pi}{2}.\ \ x = \frac{5\pi}{12} + \frac{n\pi}{2}$