MATH 180 FINAL REVIEW

For Problems # 1 – 8, find the limit if it exists:

1. $\lim_{x\to 3} \frac{\sqrt{x+6}}{x+2}$

   a. $\frac{1}{2}$ b. $-\sqrt{3}$ c. $\frac{9}{5}$ d. $\frac{3}{5}$

2. $\lim_{x\to 2} \frac{x^2+2x-8}{x^2-x-2}$

   a. $Does\ not\ exist$ b. $\frac{1}{2}$ c. 2 d. ∞

3. $\lim_{x\to 8^+} \frac{|x-8|}{x-8}$

   a. -1 b. 1 c. $Does\ not\ exist$ d. ∞

4. $\lim_{x\to 4^-} \frac{\sqrt{x}-2}{x-4}$

   a. $\frac{1}{4}$ b. 1 c. $\frac{1}{6}$ d. $Does\ not\ exist$

5. $\lim_{x\to 0} \frac{Sin\ 2x}{Sin\ x}$

   a. 1 b. 2 c. 0 d. $Does\ not\ exist$

6. $\lim_{\theta\to 0} \frac{Cos\theta Tan\theta}{\theta}$

   a. 0 b. ∞ c. 1 d. $Does\ not\ exist$

7. $\lim_{x\to -\infty} \frac{x}{\sqrt{x^2+1}}$

   a. 1 b. ∞ c. $-\infty$ d. -1

8. $\lim_{n\to\infty} \frac{n^3-4}{n^2+1}$

a. -4 b. 1 c. -1 d. $Does\ not\ exist$

9. Find any *x values* where $f(x)$ is **not** continuous. $f(x) = \frac{x+2}{x^2-2x-8}$

a. 4 b. -4, 2 c. -2, 4 d. -4, -2, 2

10. For any discontinuities found in problem #9, identify whether they are removable or non-removable.

a. $Both\ Removable$ b. $Both\ Non-Removable$

c. $x = -2 \quad Non-Removable$
$x = 4 \quad Removable$

d. $x = -2 \quad Removable$
$x = 4 \quad Non-Removable$

11. A function $f(x)$ is continuous at *c* if

a. $f(c) is\ defined$

b. $\lim_{x\to c} f(x)$ exists

c. $\lim_{x\to c} f(x) = f(c)$

d. All of the above

12. Find the slope of the tangent line to the graph of the function at the given point.

$f(x) = x^2 + 4x \qquad (1,5)$

a. 6 b. 5 c. 14 d. $-\frac{1}{6}$

13. Find an equation of the line that is tangent to the graph of the function at the given point.

$f(x) = \sqrt{x-1} \qquad (5,2)$

a. $x - 4y = 3$ b. $4x - y = -3$ c. $x - 4y = -3$ d. $4x - y = 18$

For Problems # 14 – 17, find the derivative $(f'(x))$ of the function and evaluate if requested.

14. $f(x) = x\,(2x-5)^3$

a. $(2x-5)^2(8x-5)$ b. $3x(2x-5)^2$

c. $6x(2x-5)^2$ d. $5(x-1)(2x-5)^2$

15. $f(x) = \dfrac{Cos\,x}{Csc\,x}$

a. $Cos\,2x$ b. $Sin\,2x$ c. 1 d. $\cos^{2x} + \sin^2 x$

16. $f(x) = \tan^2 x$ Evaluate at the point $\left(\frac{\pi}{4}, 1\right)$.

a. 2 b. 1 c. 4 d. $\frac{1}{2}$

17. $f(x) = \dfrac{x}{\sqrt{x^2+1}}$ Evaluate at the point $\left(1, \frac{\sqrt{2}}{2}\right)$

a. $\frac{\sqrt{2}}{2}$ b. $\frac{\sqrt{2}}{4}$ c. $2\sqrt{2}$ d. $\frac{1}{2}$

For Problems # 18 – 19, use Implicit Differentiation to find $\dfrac{dy}{dx}$.

18. $\sqrt{x\,y} = x^2y + 1$

a. $\dfrac{-2}{\sqrt{xy}-4xy}$ b. $\dfrac{4xy\sqrt{xy}-y}{x-2x^2\sqrt{xy}}$ c. $\dfrac{4xy}{x-2x^2}$ d. $\dfrac{4xy\sqrt{xy}-1}{1-2x^2\sqrt{xy}}$

19. $4xy + \ln(x^2y) = 7$

a. $\dfrac{-2y(2xy-1)}{x(4xy+1)}$ b. $\dfrac{-4xy^2-2y}{4x^2y+x}$ c. $\dfrac{xy}{2+4xy}$ d. $\dfrac{-6xy^2}{4x^2y+1}$

For Problems # 20 – 24, Differentiate -

20. $\dfrac{d}{dx}\left[\dfrac{1}{x^2+4}\right]$

a. $\dfrac{-2x}{(x^2+4)^2}$ b. $\dfrac{1}{2x}$ c. $\dfrac{-2x}{x^2+4}$ d. $\dfrac{2x}{(x^2-4)^2}$

21. $\frac{d}{dt}\ [(t^2-6)^3]$

a. $3t^2-18$ b. $3(t^2-6)^2$ c. $6t(t^2-6)^2$ d. $3(t^2-6)^2(2t-1)$

22. $f(x)=ln\sqrt{x^2-4}$

a. $\frac{1}{x^2-4}$ b. $\frac{x}{x^2-4}$ c. $\frac{2x}{\sqrt{x^2-4}}$ d. $\frac{1}{2\,(x^2-4)}$

23. $f(x)=x\,e^{2x}$

a. $e^{2x}(x+1)$ b. $e^{2x}(x+2)$ c. $2xe^{2x}$ d. $e^{2x}(2x+1)$

24. $f(x)=\ 5^{-4x}$

a. $-20(5^{-4x})$ b. $\frac{-4\,(5^{-4x})}{ln5}$ c. $\frac{-4ln5}{625^x}$ d. $-4(ln5)5^{4x}$

25. An isosceles triangle has two sides of equal length *s* and an included angle θ. If the angle θ is increasing at a rate of $\frac{1}{2}\ radian\ per\ minute$, find the rate of change of the *Area* of the triangle when $\theta=\ \frac{\pi}{6}$. Use the following formula for the Area of the triangle: $A=\frac{s^2}{2}sin\theta$.

a. $\frac{s^2}{8}$ b. $\frac{\sqrt{3}\,s^2}{8}$ c. $\frac{\sqrt{3}\,s^2}{4}$ d. $\frac{\sqrt{3}\,s}{8}$

26. A spherical balloon is inflated with helium at a rate of 800 cm^3/min . How fast is the *radius* of the balloon changing at the instant the radius is 60 cm?

a. $\frac{10}{3\pi}\ cm/min$ b. $\frac{\pi}{18}cm/min$ c. $\frac{10}{9\pi}cm/min$ d. $\frac{1}{18\pi}\ cm/min$

For Problems # 27 – 28, find the critical numbers

27. $f(x)=\ \frac{x^2-3x-4}{x-2}$

a. $x=2$ b. $x=2,4,6$ c. $x=3,4$ d. $x=-6,-2,4$

28. $f(x) = (x+2)^2(x-1)$

a. $x = -2, 1$ b. $x = -2, 0, 1$ c. $x = -2$ d. $x = -2, 0$

In Problems # 29 – 30, find the indicated absolute extrema on the given interval.

29. $y = 2x^3 - 6x \quad [0, 3]$ Absolute *maximum*

a. $(3, 36)$ b. $(1, -4)$ c. $(3, 48)$ d. $(-1, 4)$

30. $y = 3\,cosx \quad [0, 2\pi]$ Absolute *minimum*

a. $(2\pi, 3)$ b. $(1, -3)$ c. $(0, 3)\ and\ (2\pi, 3)$ d. $(\pi, -3)$

In Problems # 31 – 32, find the intervals where the function is increasing or decreasing as indicated.

31. $f(x) = -3x^2 - 4x - 2 \quad (increasing)$

a. $(-\infty, \frac{2}{3}]$ b. $[\frac{3}{2}, \infty)$ c. $(-\infty, -\frac{2}{3})$ d. $(-\frac{2}{3}, \infty)$

32. $f(x) = \frac{x}{x-5} \quad (decreasing)$

a. $(-\infty, \infty)$ b. $(-\infty, 5)$ c. $(-\infty, 5), (5, \infty)$ d. $(5, \infty)$

In Problems # 33 – 34, find the intervals where the function is concave up or concave down as indicated.

33. $f(x) = -3x^4 - x + 4 \quad (Concave\ Down)$

a. $(-\infty, 0)$ b. $(-\infty, \infty)$ c. $(0, \infty)$ d. $(-\infty, 0), (0, \infty)$

34. $f(x) = x + 2cosx \quad [0, 2\pi] \quad (Concave\ Up)$

a. $(\frac{7\pi}{6}, \frac{11\pi}{6})$ b. $(\frac{\pi}{2}, \frac{3\pi}{2})$ c. $(0, \pi)$ d. $(0, \frac{\pi}{2}), (\frac{3\pi}{2}, 2\pi)$

35. Find the Vertical Asymptote(s), if any, of the following function. $g(x) = \frac{6x}{36-x^2}$

a. $x = -6, x = 6$ b. $y = 0$ c. $x = 0$ d. $y = -6,\ y = 6$

36. Which limit should be used to find the Horizontal Asymptote(s), if any, of the following function?

$h(x) = \dfrac{5x^2-2}{x^2}$

a. $\lim\limits_{x\to\infty} h(x)$ b. $\lim\limits_{x\to-\infty} h(x)$ c. Neither a or b d. Both a and b

In Problems # 37 – 41, Evaluate the Indefinite Integrals.

37. $\int (\sqrt[4]{x^3} + 1)\, dx$

a. $\frac{4}{7}\, x^{\frac{7}{4}} + x + C$ b. $\frac{3}{7}\, x^{\frac{7}{3}} + x + C$ c. $\frac{7}{4}\, x^{\frac{7}{4}} + x + C$ d. $\frac{3}{4} x^{\frac{-1}{4}} + C$

38. $\int \dfrac{6x^2}{(4x^3-9)^3}\ dx$

a. $\dfrac{(4x^3-9)^4}{2} + C$ b. $\dfrac{-1}{4(4x^3-9)^2} + C$ c. $\dfrac{-1}{(4x^3-9)^2} + C$ d. $\dfrac{4x}{(4x^3-9)^2} + C$

39. $\int \dfrac{e^x - e^{-x}}{e^x + e^{-x}}\ dx$

a. $ln|e^x + e^{-x}| + C$ b. $ln|e^x - e^{-x}| + C$ c. $\dfrac{(e^x+e^{-x})^2}{2} + C$ d. $e^x + e^{-x} + C$

40. $\int 2^{sinx}\, cosx\, dx$

a. $\dfrac{2^{sinx}}{ln2} + C$ b. $ln2\left(2^{sinx}\right) + C$ c. $\dfrac{2^{cosx}}{ln2} + C$ d. $ln2^{sinx} + C$

41. $\int \dfrac{2x-5}{x^2+2x+2}\ dx$

a. $ln|x^2 + 2x + 2| - 3x + C$ b. $2\arctan(x+1) + C$

c. $\dfrac{1}{x+2} + C$ d. $ln|x^2 + 2x + 2| - 7\arctan(x+1) + C$

In Problems #42 – 46, Evaluate the Definite Integrals.

42. $\int_{-1}^{1} \frac{x^2+2x+1}{x^4}\, dx$

a. -2 b. -4 c. $-\frac{8}{3}$ d. 0

43. $\int_0^5 |2x-5|\, dx$

a. 25 b. $\frac{25}{2}$ c. -25 d. $-\frac{25}{2}$

44. $\int_0^{\pi}(2+cosx)dx$

a. 2π b. $2\pi-1$ c. $2\pi+1$ d. $\frac{\pi}{2}$

45. $\int_{ln2}^{ln4} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\, dx$

a. $\frac{3}{16}$ b. $\arcsin(4)-arcsin(2)$ c. $\frac{\pi}{3}-\arccos\left(\frac{1}{4}\right)$ d. $\frac{\pi}{6}-\arcsin\left(\frac{1}{4}\right)$

46. $\int_0^{\frac{\pi}{2}} \frac{cosx}{1+\sin^2 x}$ dx

a. $\frac{\pi}{4}$ b. 0 c. $-\frac{\pi}{4}$ d. $ln2$

47. Find the average value of the function over the given interval. $f(x)=cosx$ $\left[-\frac{\pi}{3},\frac{\pi}{3}\right]$

a. $\sqrt{3}$ b. $\frac{3\sqrt{3}}{2\pi}$ c. 0 d. $\frac{3}{2\pi}$

48. Assume the acceleration of an object is given by $a(t)=-32\frac{ft}{s^2}$. With what initial velocity must a ball be thrown upward from ground level to reach the top of the Washington Monument? (Approx. 550 ft.)

a. $137.0\ \frac{ft}{s}$ b. $34.3\ \frac{ft}{s}$ c. $187.6\frac{ft}{s}$ d. $98.4\frac{ft}{s}$

49. Determine the area of the region bounded by the graphs of $y = x^2 + 2$ and $y = 2x + 2$.

a. $\frac{20}{3}$ b. $\frac{8}{3}$ c. $\frac{4}{3}$ d. $-\frac{4}{3}$

50. Given that $g(x)$ is an *even* function and $\int_0^3 g(x) = 6$, find $\int_{-3}^3 g(x)$

a. 0 b. 12 c. 3 d. Not enough information

ANSWER KEY

1.	d	18.	b	35.	a
2.	c	19.	b	36.	d
3.	b	20.	a	37.	a
4.	a	21.	c	38.	b
5.	b	22.	b	39.	a
6.	c	23.	d	40.	a
7.	d	24.	c	41.	d
8.	d	25.	b	42.	c
9.	c	26.	d	43.	b
10.	d	27.	a	44.	a
11.	d	28.	d	45.	d
12.	a	29.	a	46.	a
13.	c	30.	d	47.	b
14.	a	31.	c	48.	c
15.	a	32.	c	49.	c
16.	c	33.	b	50.	b
17.	b	34.	b		