Math 130 Final Review (Updated Fall 2025)

Part I: Multiple Choice: Choose the best possible answer.

1. Given $\cos u = -\frac{2}{7}$ with u in Quadrant II, find $\cos\frac{u}{2}$ and $\sin 2u$

a. $\cos\frac{u}{2} = -\frac{\sqrt{70}}{14}$ $\quad \sin 2u = \frac{6\sqrt{5}}{7}$

b. $\cos\frac{u}{2} = \frac{\sqrt{70}}{14}$ $\quad \sin 2u = -\frac{6\sqrt{5}}{7}$

c. $\cos\frac{u}{2} = \frac{3\sqrt{14}}{14}$ $\quad \sin 2u = -\frac{12\sqrt{5}}{49}$

d. $\cos\frac{u}{2} = \frac{\sqrt{70}}{14}$ $\quad \sin 2u = -\frac{12\sqrt{5}}{49}$

For Problems 2 and 3, let $\sin A = -\frac{7}{25}$ with A in Quadrant III and $\cos B = -\frac{4}{5}$ with B in Quadrant III.

2. Find $\sin(A + B)$

a. $-\frac{4}{5}$ $\quad$ b. $\frac{3}{5}$ $\quad$ c. $\frac{4}{5}$ $\quad$ d. $-\frac{3}{5}$

3. Find $\tan(A - B)$

a. $\frac{100}{117}$ $\quad$ b. $-\frac{44}{75}$ $\quad$ c. $\frac{44}{75}$ $\quad$ d. $-\frac{44}{117}$

4. Simplify the trigonometric expression: $\frac{\sec\theta - 1}{1 - \cos\theta}$

a. $\sec\theta$ $\quad$ b. $\cos\theta$ $\quad$ c. $\frac{\sec\theta + \cos\theta}{\sin^2\theta}$ $\quad$ d. -1

5. Simplify the trigonometric expression: $\frac{1}{\cos x + 1} + \frac{1}{\cos x - 1}$

a. $\sec x$ $\quad$ b. $-2\csc x \cot x$ $\quad$ c. $-2\csc^2 x$ $\quad$ d. $\frac{2}{\cos^2 x - 1}$

6. What is the domain of $\sin^{-1} x$?

a. $(-\infty, \infty)$ $\quad$ b. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\quad$ c. $[-1, 1]$ $\quad$ d. $[0, \pi]$

7. Given $\sin\theta = -\frac{12}{15}$ and $\tan\theta < 0$, find the five remaining trigonometric functions of θ.

a. $\csc\theta = -\frac{15}{12}$
$\cos\theta = \frac{9}{15}$
$\sec\theta = \frac{15}{9}$
$\tan\theta = -\frac{12}{9}$
$\cot\theta = -\frac{9}{12}$

b. $\csc\theta = \frac{15}{12}$
$\cos\theta = -\frac{9}{15}$
$\sec\theta = -\frac{15}{9}$
$\tan\theta = -\frac{12}{9}$
$\cot\theta = -\frac{9}{12}$

c. $\csc\theta = -\frac{15}{12}$
$\cos\theta = -\frac{9}{15}$
$\sec\theta = -\frac{15}{9}$
$\tan\theta = \frac{12}{9}$
$\cot\theta = \frac{9}{12}$

d. $\csc\theta = \frac{15}{12}$
$\cos\theta = \frac{9}{15}$
$\sec\theta = \frac{15}{9}$
$\tan\theta = \frac{12}{9}$
$\cot\theta = \frac{9}{12}$

8. Identify the amplitude, period, horizontal shift, and vertical shift for the following function:

$f(x) = 1 - 3\sin(2x + \pi)$.

a. Amp $= 3$
Per $= 2\pi$
HS $= left\ \frac{\pi}{2}$
VS $= up\ 1$

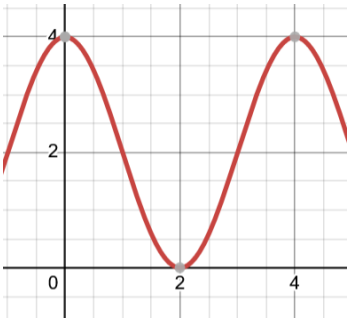
b. Amp $= 3$
Per $= \pi$
HS $= right\ \frac{\pi}{2}$
VS $= up\ 1$

c. Amp $= -3$
Per $= \pi$
HS $= left\ \frac{\pi}{2}$
VS $= down\ 1$

d. Amp $= 3$
Per $= \pi$
HS $= left\ \frac{\pi}{2}$
VS $= up\ 1$

9. Find the equation that matches the graph:

a. $y = 2 + 2\cos\left(\frac{\pi x}{2}\right)$
b. $y = 2 + 2\cos(2\pi x)$
c. $y = 4 + \cos\left(\frac{\pi x}{2}\right)$
d. $y = 2 + 2\cos\left(\frac{2x}{2}\right)$



10. Evaluate $\tan^{-1}(-1)$

a. $\frac{3\pi}{4}, \frac{7\pi}{4}$ b. $\frac{7\pi}{4}$ c. $-\frac{\pi}{4}$ d. $\frac{3\pi}{4}$

11. Evaluate $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

a. $\frac{\pi}{4}, \frac{7\pi}{4}$ b. $-\frac{\pi}{4}$ c. $\frac{3\pi}{4}$ d. $\frac{\pi}{4}$

12. Evaluate $\sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$

a. $\frac{\sqrt{3}}{2}$ b. $-\frac{\sqrt{3}}{2}$ c. $\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$ d. $\frac{\sqrt{3}\pi}{2}$

13. Evaluate $\sec\left(\tan^{-1}\left(\frac{1}{2x}\right)\right)$

a. $\sqrt{4x^2+1}$ b. $\frac{\sqrt{4x^2+1}}{2x}$ c. $\frac{\sqrt{2x^2+1}}{2x}$ d. $\frac{2x}{\sqrt{4x^2+1}}$

14. In triangle ABC, if $a = 3.7\ cm, c = 6.4\ cm, and\ B = 23°$, find b.

a. $4.1\ cm$ b. $3.3\ cm$ c. $5.7\ cm$ d. $11.1\ cm$

15. In triangle ABC, if $B = 110°, C = 40°, and\ b = 18.0\ inches$, find a.

a. $9.6\ in$ b. $12.3\ in$ c. $33.8\ in$ d. $Not\ enough\ info$

16. In triangle ABC, if $a = 4.8\ in, b = 6.3\ in, and\ c = 7.5\ in$, find the area of the triangle.

a. $4.9\ in^2$ b. $15.0\ in^2$ c. $45.9\ in^2$ d. $18.0\ in^2$

17. In triangle ABC, if $B = 57°, a = 7.3\ m, and\ c = 3.8\ m$, find the area of the triangle.

a. $43.4\ m^2$ b. $23.0\ m^2$ c. $46.0\ m^2$ d. $11.6\ m^2$

18. Give an angle between $0°$ and $360°$ coterminal with the angle $475°$.

a. $15°$ b. $215°$ c. $115°$ d. $205°$

19. Given central angle $\theta = \frac{3\pi}{4}$ and radius $4\ in$, find the arc length, s.

a. $\frac{3\pi}{16}\ in$ b. $12\pi\ in$ c. $3\pi\ in$ d. $6\pi\ in$

20. Given central angle $\theta = 72°$ and radius $5\ m$, find the area of the sector of the circle.

a. $5\pi\ m^2$ b. $10\pi\ m^2$ c. $900\pi\ m^2$ d. $\frac{\pi}{5}\ m^2$

21. Convert $\theta = 435°$ to radians.

a. $\frac{7\pi}{12}$ b. $\frac{29\pi}{12}$ c. $\frac{\pi}{4}$ d. $\frac{78300}{\pi}$

22. Find $\cos 45°$.

a. $\frac{1}{2}$ b. 1 c. $\frac{\sqrt{3}}{2}$ d. $\frac{\sqrt{2}}{2}$

23. Find $\cot 90°$

a. $\sqrt{3}$ b. $undefined$ c. $\frac{\sqrt{3}}{3}$ d. 0

24. Find $\csc 60°$

a. 2 b. $\frac{2\sqrt{3}}{3}$ c. $\frac{\sqrt{3}}{2}$ d. $\frac{1}{2}$

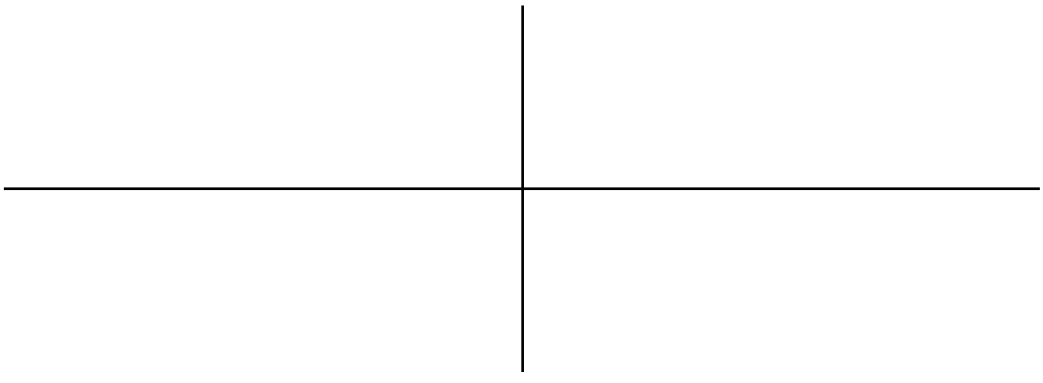
Part II. Short Answer Section: Show your work.

1. Two straight wires are strung on opposite sides of a tent pole and anchored to the ground by two stakes. One of the wires is $56\ ft$ long and makes an angle of $47°$ with the ground. The other wire is $65\ ft$ long and makes an angle of $37°$ with the ground. How far apart are the stakes that hold the wires to the ground?

2. Given the following trigonometric function: $y = -1 + \frac{1}{3}\tan\left(2x - \frac{\pi}{2}\right)$

   a. Find the period, amplitude, horizontal translation, and vertical translation.

   b. Graph (at least one period):



3. Prove the following:

a. $\cot x - \tan x = \frac{\cos 2x}{\sin x \cos x}$

b. $\sin(60° + x) + \sin(60° - x) = \sqrt{3}\cos x$

c. $(1 - \sin\theta)(1 + \sin\theta) = \cos^2\theta$

d. $\cot A = \frac{\sin 2A}{1 - \cos 2A}$

4. Solve the trigonometric equations:

a. $\csc^2 x + 3\csc x - 4 = 0$ over $[0\,, 2\pi)$

b. $2\sin^2 x + 5\cos x - 4 = 0$

c. $2\sin 2x + \sqrt{3} = 0$ over $[0\,, 2\pi)$

d. $\sec 4x - 2 = 0$

5. Given $\boldsymbol{u} = \langle -2\,, 5\rangle$ and $\boldsymbol{v} = \langle -1\,, -8\rangle$, find the following:

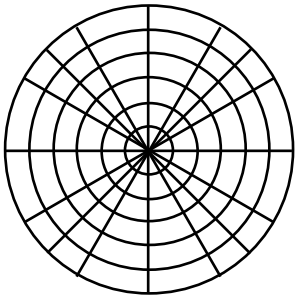
   a. The magnitude and direction angle of vector $\boldsymbol{u}$

   b. The magnitude and direction angle of vector $\boldsymbol{v}$

   c. The dot product $\boldsymbol{u} \cdot \boldsymbol{v}$

   d. The measure of the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ to the nearest tenth of a degree.

6. Plot the polar coordinates $\left(-3\,, \frac{5\pi}{6}\right)$ on the polar graph below. Then convert to rectangular coordinates.



7. Convert $(1, -\sqrt{3})$ to polar coordinates. Give two possible solutions with θ between 0 and 2π.

8. Derive the power reducing formula for $\cos^2 x$ using the one of the double angle identities for cosine.

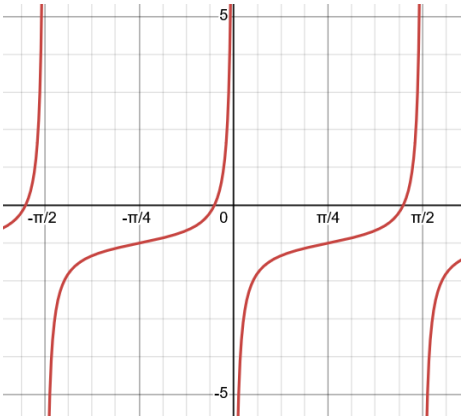
9. A surveyor walking a property line walks $3.2\ miles$ in the direction of $N\ 25°\ W$. They then turn $90°$ and walk $4.3\ miles$ in the direction $N\ 65°\ E$. At that time, how far are they from their starting point?

Answer Key

1. D	7. A	13. B	19. C
2. C	8. D	14. B	20. A
3. D	9. A	15. A	21. B
4. A	10. C	16. B	22. D
5. B	11. C	17. D	23. D
6. C	12. A	18. C	24. B

Additional Problem Answers

1. $90.1\ ft$
2. Period: $\frac{\pi}{2}$ Amp: $none$ HT: $right\ \frac{\pi}{4}$ VT: $down\ 1$



3. Proofs:
   a. $\cot x - \tan x = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$

$$= \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x} - \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x}$$

$$= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

$$= \frac{\cos 2x}{\sin x \cos x}$$

b. $\sin(60° + x) + \sin(60° - x) = \sin 60° \cos x + \cos 60° \sin x + \sin 60° \cos x - \cos 60° \sin x$

$$= \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$$

$$= \frac{2\sqrt{3}}{2}\cos x$$

$$= \sqrt{3}\cos x$$

c. $(1 - \sin\theta)(1 + \sin\theta) = 1 + \sin\theta - \sin\theta - \sin^2\theta$

$$= 1 - \sin^2\theta$$

$$= \cos^2\theta$$

d. $\frac{\sin 2A}{1-\cos 2A} = \frac{2\sin A\cos A}{1-(1-2\sin^2 A)}$

$$= \frac{2\sin A\cos A}{2\sin^2 A}$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A$$

4. a. $x = \frac{\pi}{2}, 3.39, 6.54$

b. $\frac{\pi}{3} + 2\pi k, \frac{5\pi}{3} + 2\pi k$

c. $x = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

d. $\frac{\pi}{12} + \frac{\pi}{2}k, \frac{5\pi}{12} + \frac{\pi}{2}k$

5. a. $|\boldsymbol{u}| = \sqrt{29}, \theta = 111.8°$

b. $|\boldsymbol{v}| = \sqrt{65}, \theta = 262.9°$

c. -38

d. $\theta = 151.1°$

6. $\left(\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

7. $\left(2\,,\frac{5\pi}{3}\right)\,,\left(-2\,,\frac{2\pi}{3}\right)$

8. $\cos^2 x = 1 - \sin^2 x$

$+\cos^2 x \qquad + \cos^2 x$

$2\cos^2 x = 1 - \sin^2 x + \cos^2 x$

$2\cos^2 x = 1 + (\cos^2 x - \sin^2 x)$

$2\cos^2 x = 1 + (\cos 2x)$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

9. $5.4\ miles$