Integration Table

Forms Involving u^n:

1. $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, n \neq -1$

2. $\int \frac{1}{u} \, du = \ln|u| + C$

Forms Involving $a + bu$:

3. $\int \frac{u}{a+bu} \, du = \frac{1}{b^2}(bu - a\ln|a+bu|) + C$

4. $\int \frac{u}{(a+bu)^2} \, du = \frac{1}{b^2}\left(\frac{a}{a+bu} + \ln|a+bu|\right) + C$

5. $\int \frac{u}{(a+bu)^n} \, du = \frac{1}{b^2}\left[\frac{-1}{(n-2)(a+bu)^{n-2}} + \frac{a}{(n-1)(a+bu)^{n-1}}\right] + C, n \neq 1, 2$

6. $\int \frac{u^2}{a+bu} \, du = \frac{1}{b^3}\left[-\frac{bu}{2}(2a - bu) + a^2\ln|a+bu|\right] + C$

7. $\int \frac{u^2}{(a+bu)^2} \, du = \frac{1}{b^3}\left(bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$

8. $\int \frac{u^2}{(a+bu)^3} \, du = \frac{1}{b^3}\left[\frac{2a}{a+bu} - \frac{a^2}{2(a+bu)^2} + \ln|a+bu|\right] + C$

9. $\int \frac{u^2}{(a+bu)^n} \, du = \frac{1}{b^3}\left[\frac{-1}{(n-3)(a+bu)^{n-3}} + \frac{2a}{(n-2)(a+bu)^{n-2}} - \frac{a^2}{(n-1)(a+bu)^{n-1}}\right] + C, n \neq 1, 2, 3$

10. $\int \frac{1}{u(a+bu)} \, du = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

11. $\int \frac{1}{u(a+bu)^2} \, du = \frac{1}{a}\left(\frac{1}{a+bu} + \frac{1}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

12. $\int \frac{1}{u^2(a+bu)} \, du = -\frac{1}{a}\left(\frac{1}{u} + \frac{b}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

13. $\int \frac{1}{u^2(a+bu)^2} \, du = -\frac{1}{a^2}\left[\frac{a+2bu}{u(a+bu)} + \frac{2b}{a}\ln\left|\frac{u}{a+bu}\right|\right] + C$

Forms Involving $a + bu + cu^2, b^2 \neq 4ac$

14. $\int \frac{1}{a+bu+cu^2} \, du = \begin{cases} \frac{2}{\sqrt{4ac-b^2}}\arctan\frac{2cu+b}{\sqrt{4ac-b^2}} + C, & b^2 < 4ac \\ \frac{1}{\sqrt{b^2-4ac}}\ln\left|\frac{2cu+b-\sqrt{b^2-4ac}}{2cu+b+\sqrt{b^2-4ac}}\right| + C, & b^2 > 4ac \end{cases}$

15. $\int \frac{u}{a+bu+cu^2} \, du = \frac{1}{2c}\left(\ln|a+bu+cu^2| - b\int\frac{1}{a+bu+cu^2} \, du\right)$

Forms Involving $\sqrt{a+bu}$

16. $\int u^n\sqrt{a+bu} \, du = \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu} \, du\right]$

17. $\int \frac{1}{u\sqrt{a+bu}} \, du = \begin{cases} \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C, & a > 0 \\ \frac{2}{\sqrt{-a}}\arctan\sqrt{\frac{a+bu}{-a}} + C, & a < 0 \end{cases}$

18. $\int \frac{1}{u^n\sqrt{a+bu}} \, du = \frac{-1}{a(n-1)}\left[\frac{\sqrt{a+bu}}{u^{n-1}} + \frac{(2n-3)b}{2}\int\frac{1}{u^{n-1}\sqrt{a+bu}} \, du\right], n \neq 1$

19. $\int \frac{\sqrt{a+bu}}{u} \, du = 2\sqrt{a+bu} + a\int\frac{1}{u\sqrt{a+bu}} \, du$

20. $\int \frac{\sqrt{a+bu}}{u^n} \, du = \frac{-1}{a(n-1)}\left[\frac{(a+bu)^{3/2}}{u^{n-1}} + \frac{(2n-5)b}{2}\int\frac{\sqrt{a+bu}}{u^{n-1}} \, du\right], n \neq 1$

21. $\int \frac{u}{\sqrt{a+bu}} \, du = \frac{-2(2a-bu)}{3b^2}\sqrt{a+bu} + C$

22. $\int \frac{u^n}{\sqrt{a+bu}} \, du = \frac{2}{(2n+1)b}\left(u^n\sqrt{a+bu} - na\int\frac{u^{n-1}}{\sqrt{a+bu}} \, du\right)$

Forms Involving $a^2 \pm u^2, \ a > 0$

23. $\int \frac{1}{a^2+u^2} \, du = \frac{1}{a}\arctan\frac{u}{a} + C$

24. $\int \frac{1}{u^2-a^2} \, du = -\int\frac{1}{a^2-u^2} \, du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

25. $\int \frac{1}{(a^2\pm u^2)^n} \, du = \frac{1}{2a^2(n-1)}\left[\frac{u}{(a^2\pm u^2)^{n-1}} + (2n-3)\int\frac{1}{(a^2\pm u^2)^{n-1}} \, du\right], n \neq 1$

Forms Involving $\sqrt{u^2 \pm a^2}, \ a > 0$

26. $\int \sqrt{u^2 \pm a^2} \, du = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \pm a^2\ln\left|u + \sqrt{u^2 \pm a^2}\right|\right) + C$

27. $\int u^2\sqrt{u^2 \pm a^2} \, du = \frac{1}{8}\left[u(2u^2 \pm a^2)\sqrt{u^2 \pm a^2} - a^4\ln\left|u + \sqrt{u^2 \pm a^2}\right|\right] + C$

28. $\int \frac{\sqrt{u^2+a^2}}{u} \, du = \sqrt{u^2+a^2} - a\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

29. $\int \frac{\sqrt{u^2-a^2}}{u} \, du = \sqrt{u^2-a^2} - a\arcsec\frac{|u|}{a} + C$

30. $\int \frac{\sqrt{u^2\pm a^2}}{u^2} \, du = \frac{-\sqrt{u^2\pm a^2}}{u} + \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

31. $\int \frac{1}{\sqrt{u^2\pm a^2}} \, du = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

32. $\int \frac{1}{u\sqrt{u^2+a^2}} \, du = \frac{-1}{a}\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

33. $\int \frac{1}{u\sqrt{u^2-a^2}} \, du = \frac{1}{a}\arcsec\frac{|u|}{a} + C$

34. $\int \frac{u^2}{\sqrt{u^2\pm a^2}} \, du = \frac{1}{2}\left(u\sqrt{u^2 \pm a^2} \mp a^2\ln\left|u + \sqrt{u^2 \pm a^2}\right|\right) + C$

35. $\int \frac{1}{u^2\sqrt{u^2\pm a^2}} \, du = \mp\frac{\sqrt{u^2\pm a^2}}{a^2u} + C$

36. $\int \frac{1}{(u^2\pm a^2)^{3/2}} \, du = \frac{\pm u}{a^2\sqrt{u^2\pm a^2}} + C$

Forms Involving $\sqrt{a^2 - u^2}, \ a > 0$

37. $\int \sqrt{a^2-u^2} \, du = \frac{1}{2}\left(u\sqrt{a^2-u^2} + a^2\arcsin\frac{u}{a}\right) + C$

38. $\int u^2\sqrt{a^2-u^2} \, du = \frac{1}{8}\left[u(2u^2-a^2)\sqrt{a^2-u^2} + a^4\arcsin\frac{u}{a}\right] + C$

39. $\int \frac{\sqrt{a^2-u^2}}{u} \, du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

40. $\int \frac{\sqrt{a^2-u^2}}{u^2} \, du = \frac{-\sqrt{a^2-u^2}}{u} - \arcsin\frac{u}{a} + C$

41. $\int \frac{1}{\sqrt{a^2-u^2}} \, du = \arcsin\frac{u}{a} + C$

42. $\int \frac{1}{u\sqrt{a^2-u^2}} \, du = \frac{-1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

43. $\int \frac{u^2}{\sqrt{a^2-u^2}}\, du = \frac{1}{2}\left(-u\sqrt{a^2-u^2} + a^2 \arcsin\frac{u}{a}\right) + C$

44. $\int \frac{1}{u^2\sqrt{a^2-u^2}}\, du = \frac{-\sqrt{a^2-u^2}}{a^2 u} + C$

45. $\int \frac{1}{(a^2-u^2)^{3/2}}\, du = \frac{u}{a^2\sqrt{a^2-u^2}} + C$

Forms Involving $\sin u$ or $\cos u$

46. $\int \sin u\, du = -\cos u + C$

47. $\int \cos u\, du = \sin u + C$

48. $\int \sin^2 u\, du = \frac{1}{2}(u - \sin u\, \cos u) + C$

49. $\int \cos^2 u\, du = \frac{1}{2}(u + \sin u\, \cos u) + C$

50. $\sin^n u\, du = -\frac{\sin^{n-1} u \cos u}{n} + \frac{n-1}{n}\int \sin^{n-2} u\, du$

51. $\int \cos^n u\, du = \frac{\cos^{n-1} u \sin u}{n} + \frac{n-1}{n}\int \cos^{n-2} u\, du$

52. $\int u\, \sin u\, du = \sin u - u\cos u + C$

53. $\int u\cos u\, du = \cos u + u\sin u + C$

54. $\int u^n \sin u\, du = -u^n \cos u + n\int u^{n-1}\cos u\, du$

55. $\int u^n \cos u\, du = u^n \sin u - n\int u^{n-1}\sin u\, du$

56. $\int \frac{1}{1\pm\sin u}\, du = \tan u \mp \sec u + C$

57. $\int \frac{1}{1\pm\cos u}\, du = -\cot u \pm \csc u + C$

58. $\int \frac{1}{\sin u\cos u}\, du = \ln|\tan u| + C$

Forms Involving $\tan u, \cot u, \sec u, \csc u$

59. $\int \tan u\, du = -\ln|\cos u| + C$

60. $\int \cot u\, du = \ln|\sin u| + C$

61. $\int \sec u\, du = \ln|\sec u + \tan u| + C$

62. $\int \csc u\, du = \ln|\csc u - \cot u| + C$

63. $\int \tan^2 u\, du = -u + \tan u + C$

64. $\int \cot^2 u\, du = -u - \cot u + C$

65. $\int \sec^2 u\, du = \tan u + C$

66. $\int \csc^2 u\, du = -\cot u + C$

67. $\int \tan^n u\, du = \frac{\tan^{n-1} u}{n-1} - \int (\tan^{n-2} u)\, du, n \neq 1$

68. $\int \cot^n u\, du = -\frac{\cot^{n-1} u}{n-1} - \int (\cot^{n-2} u)\, du, n \neq 1$

69. $\int \sec^n u\, du = \frac{\sec^{n-2} u \tan u}{n-1} + \frac{n-2}{n-1}\int (\sec^{n-2} u)\, du, n \neq 1$

70. $\int \csc^n u\, du = -\frac{\csc^{n-2} u \cot u}{n-1} + \frac{n-2}{n-1}\int \csc^{n-2} u\, du, n \neq 1$

71. $\int \frac{1}{1\pm\tan u}\, du = \frac{1}{2}(u \pm \ln|\cos u \pm \sin u|) + C$

72. $\int \frac{1}{1\pm\cot u}\, du = \frac{1}{2}(u \mp \ln|\sin u \pm \cos u|) + C$

73. $\int \frac{1}{1\pm\sec u}\, du = u + \cot u \mp \csc u + C$

74. $\int \frac{1}{1\pm\csc u}\, du = u - \tan u \pm \sec u + C$

Forms Involving Inverse Trigonometric Functions

75. $\int \arcsin u\, du = u\arcsin u + \sqrt{1-u^2} + C$

76. $\int \arccos u\, du = u\arccos u - \sqrt{1-u^2} + C$

77. $\int \arctan u\, du = u\arctan u - \ln\sqrt{1+u^2} + C$

78. $\int \text{arccot}\, u\, du = u\,\text{arccot}\, u + \ln\sqrt{1+u^2} + C$

79. $\int \text{arcsec}\, u\, du = u\,\text{arcsec}\, u - \ln|u + \sqrt{u^2-1}| + C$

80. $\int \text{arccsc}\, u\, du = u\,\text{arccsc}\, u + \ln|u + \sqrt{u^2-1}| + C$

Forms Involving e^u

81. $\int e^u\, du = e^u + C$

82. $\int u e^u\, du = (u-1)e^u + C$

83. $\int u^n e^u\, du = u^n e^u - n\int u^{n-1}e^u\, du$

84. $\int \frac{1}{1+e^u}\, du = u - \ln(1 + e^u) + C$

85. $\int e^{au} \sin bu\, du = \frac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu) + C$

86. $\int e^{au} \cos bu\, du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu) + C$

Forms Involving $\ln u$

87. $\int \ln u\, du = u(-1 + \ln u) + C$

88. $\int u\ln u\, du = \frac{u^2}{4}(-1 + 2\ln u) + C$

89. $\int u^n \ln u\, du = \frac{u^{n+1}}{(n+1)^2}[-1 + (n+1)\ln u] + C, n \neq -1$

90. $\int (\ln u)^2\, du = u[2 - 2\ln u + (\ln u)^2] + C$

91. $\int (\ln u)^n\, du = u(\ln u)^n - n\int (\ln u)^{n-1}\, du$