

MEASUREMENTS

A **measurement** includes a number and a unit.

3 feet

7 minutes

12 gallons

Standard units of measurement have been established to simplify trade and commerce.

TIME

Equivalences between units of time are:

60 seconds (sec) = 1 minute (min)

60 min = 1 hour (hr)

24 hr = 1 day

7 days = 1 week (wk)

52 wk = 1 year (yr)

12 months (mo) = 1 yr

365 days = 1 yr

U.S. CUSTOMARY SYSTEM OF MEASUREMENT

LENGTH

The standard U.S. Customary System units of length are **inch**, **foot**, **yard**, and **mile**.

Equivalences between units of length in the U.S. Customary System are:

12 inches (in.) = 1 foot (ft)

3 ft = 1 yard (yd)

36 in. = 1 yard (yd)

5280 ft = 1 mile (mi)

Because 3 ft = 1 yd, the conversion rates $\frac{3 \text{ ft}}{1 \text{ yd}}$ and $\frac{1 \text{ yd}}{3 \text{ ft}}$ are both equal to 1.

Examples: Convert 27 ft. to yards

$$\begin{aligned} \frac{27 \cancel{\text{ft}}}{1} \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} \\ = \frac{27 \text{ yd}}{3} = 9 \text{ yd} \end{aligned}$$

Convert 5 yd to feet

$$\begin{aligned} \frac{5 \cancel{\text{yd}}}{1} \times \frac{3 \text{ ft}}{1 \cancel{\text{yd}}} \\ = \frac{15 \text{ ft}}{1} = 15 \text{ ft} \end{aligned}$$

Note that in the conversion rate chosen, the unit in the numerator is the same as the unit desired in the answer. The unit in the denominator is the same as the unit in the given measurement.

WEIGHT

Weight is a measure of how strongly Earth is pulling on an object. The U.S. Customary System units of weight are **ounce**, **pound**, and **ton**.

Equivalences between units of weight in the U.S. Customary System are:

16 ounces (oz) = 1 pound (lb)

2000 lb = 1 ton

Because 16 oz = 1 lb, the conversion rates $\frac{16 \text{ oz}}{1 \text{ lb}}$ and $\frac{1 \text{ lb}}{16 \text{ oz}}$ are both equal to 1.

Example: Convert 62 oz to pounds.

$$\frac{62 \cancel{\text{oz}}}{1} \times \frac{1 \text{ lb}}{16 \cancel{\text{oz}}} = \frac{62 \text{ oz}}{16} = 3 \frac{7}{8} \text{ lb}$$

CAPACITY

Liquid substances are measured in units of **capacity**. The standard U.S. Customary units of capacity are the **fluid ounce, cup, pint, quart, and gallon**.

Equivalences between units of capacity in the U.S. Customary System are:

$$8 \text{ fluid ounces (fl oz)} = 1 \text{ cup (c)}$$

$$2 \text{ c} = 1 \text{ pint (pt)}$$

$$2 \text{ pt} = 1 \text{ quart (qt)}$$

$$4 \text{ qt} = 1 \text{ gallon (gal)}$$

Because $8 \text{ fl oz} = 1 \text{ c}$, the conversion rates $\frac{8 \text{ fl oz}}{1 \text{ c}}$ and $\frac{1 \text{ c}}{8 \text{ fl oz}}$ are both equal to 1.

Examples: Convert 36 fl oz to cups.

$$\begin{aligned} \frac{36 \cancel{\text{ fl oz}}}{1} \times \frac{1 \text{ c}}{8 \cancel{\text{ fl oz}}} \\ = \frac{36 \text{ c}}{8} = 4\frac{1}{2} \text{ c} \end{aligned}$$

Convert 3 qt to cups.

$$\begin{aligned} \frac{3 \cancel{\text{ qt}}}{1} \times \frac{2 \cancel{\text{ pt}}}{1 \cancel{\text{ qt}}} \times \frac{2 \text{ c}}{1 \cancel{\text{ pt}}} \\ = \frac{12 \text{ c}}{1} = 12 \text{ c} \end{aligned}$$

METRIC SYSTEM OF MEASUREMENT

LENGTH

In 1789, an attempt was made to standardize units of measurement internationally in order to simplify trade and commerce between nations. A commission in France developed a system of measurement known as the **metric system**.

The basic unit of length in the metric system is the **meter**. All units of length in the metric system are derived from the meter. Prefixes to the basic unit denote the length of each unit. For example, the prefix "centi-" means one-hundredth, so 1 centimeter is 1 one-hundredth of a meter.

kilo- = 1000	1 kilometer (km) = 1000 meters (m)
hecto- = 100	1 hectometer (hm) = 100 m
deca- = 10	1 decameter (dam) = 10 m
	1 meter = 1 m
deci- = 0.1	1 decimeter (dm) = 0.1 m
centi- = 0.01	1 centimeter (cm) = 0.01 m
milli- = 0.001	1 millimeter (mm) = 0.001 m

Conversion between units of length in the metric system involves moving the decimal point to the right or to the left. Listing the units in order from largest to smallest will indicate how many places and in which direction to move the decimal point.

For example, to convert 4200 cm to meters requires moving 2 positions to the left.

METRIC PREFIXES		
Prefix	Symbol	Meaning
Milli-	m	One thousandth $\left(\frac{1}{1000}\right)$
Centi-	c	One hundredth $\left(\frac{1}{100}\right)$
Deci-	d	One tenth $\left(\frac{1}{10}\right)$
Deca-	da	Ten (10)
Hecto-	h	One hundred (100)
Kilo-	k	One thousand (1000)

MASS

Mass and weight are closely related. Weight is a measure of how strongly Earth is pulling on an object. Therefore, an object's weight is less in space than on Earth's surface. However, the amount of material in the object, its **mass**, remains the same. On the surface of Earth, mass and weight can be used interchangeably.

The basic unit of mass in the metric system is the **gram**. If a box that is 1 cm long on each side is filled with water, then the mass of that water is 1 gram.



$1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}$

$1\text{ mg} = \text{the mass of water in the box}$

The units of mass in the metric system have the same prefixes as the units of length:

1 kilogram (kg) = 1000 grams (g)

1 hectogram (hg) = 100 g

1 decagram (dag) = 10 g

1 gram = 1 g

1 decigram (dg) = 0.1 g

1 centigram (cg) = 0.01 g

1 milligram (mg) = 0.001 g

CAPACITY

The basic unit of capacity in the metric system is the **liter**.

The units of capacity in the metric system have the same prefixes as the units of length:

1 kiloliter (kL) = 1000 liters (L)

1 hectoliter (hL) = 100 L

1 decaliter (daL) = 10 L

1 liter = 1 L

1 deciliter (dL) = 0.1 L

1 centiliter (cL) = 0.01 L

1 milliliter (mL) = 0.001 L

The milliliter is equal to 1 cubic centimeter (cm^3).

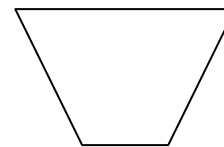
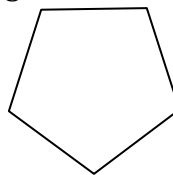
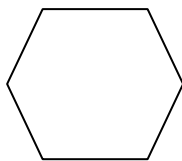
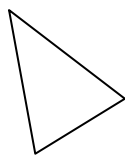
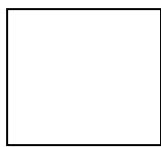


$1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}$

$1\text{ mL} = 1\text{ cm}^3$

PLANE GEOMETRIC FIGURES

A **polygon** is a closed figure determined by three or more line segments that lie in a plane. The line segments that form the polygon are called its **sides**. The figures below are examples of polygons:



A **regular polygon** is one in which each side has the same length and each angle has the same measure. The first, third, and fourth figures above are regular polygons.

The name of a polygon is based on the number of its sides. The table below lists the names of polygons that have from 3 to 10 sides:

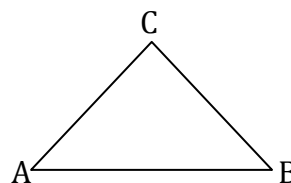
Number of Sides	Name of Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

Triangles and quadrilaterals are two of the most common types of polygons. Triangles are distinguished by the number of equal sides and also by the measures of their angles.

An **isosceles triangle** has two sides of equal length.
The angles opposite the equal sides are of equal measure.

$$AC = BC$$

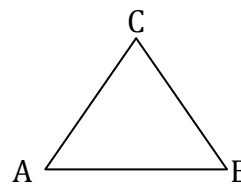
$$\angle A = \angle B$$



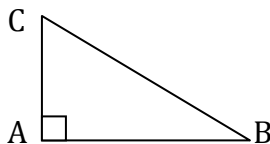
The three sides of an **equilateral triangle** are of equal length.
The three angles are of equal measure.

$$AB = BC = AC$$

$$\angle A = \angle B = \angle C$$



A **right triangle** has a right angle.



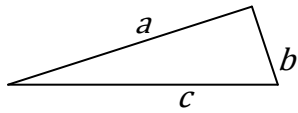
Types of Quadrilaterals	Properties
Quadrilateral	Four-sided polygon.
Square	Opposite sides are parallel. All sides are equal length. All angles measure 90° . Diagonals are equal in length.
Rectangle	Opposite sides are parallel. Opposite sides are equal length. All angles measure 90° . Diagonals are equal in length.
Parallelogram	Opposite sides are parallel. Opposite sides are equal length. Opposite angles equal in measure.
Rhombus	Opposite sides are parallel. All sides are equal length. Opposite angles equal in measure.
Trapezoid	Two sides parallel.
Isosceles Trapezoid	Two sides parallel. Nonparallel sides equal in length.

PERIMETER

The **perimeter** of a plane geometric figure is a measure of the distance around the figure.

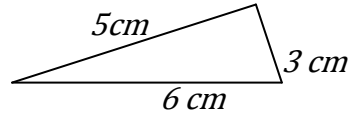
Perimeter of a Triangle

$$P = a + b + c$$



Find the perimeter of the triangle shown at the right.

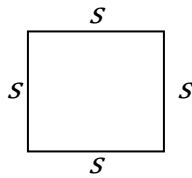
$$\begin{aligned} P &= a + b + c \\ &= 5 \text{ cm} + 3 \text{ cm} + 6 \text{ cm} \\ &= 14 \text{ cm} \end{aligned}$$



The perimeter of the triangle is 14 cm.

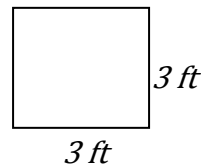
Perimeter of a Square

$$P = 4s$$



Find the perimeter of the square shown at the right.

$$\begin{aligned} P &= 4s \\ &= 4 \cdot (3 \text{ ft}) \\ &= 12 \text{ ft} \end{aligned}$$



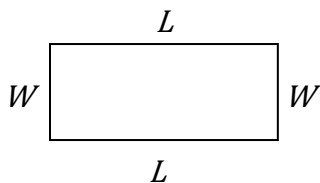
The perimeter of the square is 12 ft.

The perimeter of a quadrilateral is the sum of the lengths of the four sides.

Perimeter of a Rectangle

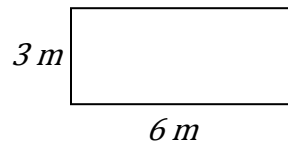
A rectangle is a quadrilateral with opposite sides of equal length. The length of a rectangle refers to the longer side and the width refers to the length of the shorter side.

$$P = 2L + 2W$$



Find the perimeter of the rectangle shown at the right.

$$\begin{aligned} P &= 2L + 2W \\ &= 2(6 \text{ m}) + 2(3 \text{ m}) \\ &= 12 \text{ m} + 6 \text{ m} \\ &= 18 \text{ m} \end{aligned}$$

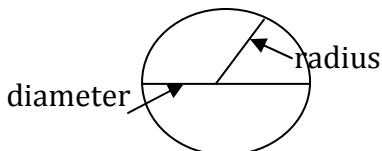


The perimeter of the rectangle is 18 m.

Circumference of a Circle

The distance around a circle is called the **circumference**. The circumference of a circle is equal to the product of π (pi) and the diameter.

$$C = \pi \cdot d \text{ or } C = 2\pi r \text{ (because diameter} = 2r)$$



The formula for circumference uses the number π (pi).

The value of π can be approximated by a fraction or a decimal.

$$\pi \approx 3.14 \quad \text{or} \quad \pi = \frac{22}{7}$$

The π key on a calculator gives a closer approximation of π than 3.14.

Perimeter of a Composite Geometric Figure

Composite geometric figures are figures made from two or more geometric figures.



Perimeter of composite figure = 3 sides of rectangle + $\frac{1}{2}$ circumference of circle

$$= 2L + W + \frac{1}{2} \pi d$$

A diagram showing a composite figure with a horizontal top side of 5m and a vertical left side of 4m. This is equal to a rectangle with a horizontal top side of 5m and a vertical left side of 4m, plus a circle with a diameter of 4m. The equation is: $4m$ [composite figure] $= 4m$ [rectangle] $+ \frac{1}{2} \pi \cdot 4m$

$$P \approx 2(5m) + 4m + \frac{1}{2}(3.14)(4m) \\ \approx 20.28 \text{ m}$$

The perimeter is approximately 20.28 m.

AREA

Area is a measure of the amount of surface in a region. Area is measured in square units.

A square that measures 1 inch on each side has an area of 1 square inch, which is written 1 in^2 .

A square that measures 1 centimeter on each side has an area of 1 square centimeter, which is written 1 cm^2 . Larger areas can be measured in square feet (ft^2), square meters (m^2), square miles (mi^2), acres ($43,560 \text{ ft}^2$), or any other square unit.

Area of a Rectangle

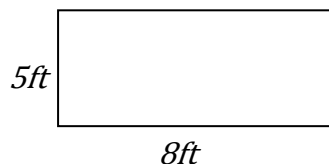
The area of a rectangle can be found by multiplying the length by its width.

$$A = LW$$

Find the area of the rectangle shown at the right.

$$A = LW \\ = (8 \text{ ft})(5 \text{ ft}) \\ = 40 \text{ ft}^2$$

The area of the rectangle is 40 ft^2 .



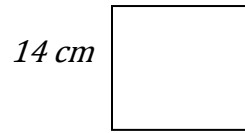
Area of a Square

A square is a rectangle in which all sides are the same length. Therefore, both the length and width can be represented by a side.

$$A = s \cdot s \quad \text{or} \quad s^2$$

Find the area of the square shown at the right.

$$\begin{aligned} A &= s^2 \\ &= (14 \text{ cm})^2 \\ &= 196 \text{ cm}^2 \end{aligned}$$



The area of the square is 196 cm^2 .

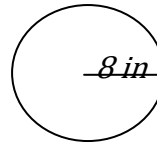
Area of a Circle

The area of a circle is equal to the product of π and the square of the radius.

$$A = \pi r^2$$

Find the area of the circle shown at the right.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(8 \text{ in})^2 \\ &\approx 3.14 (64 \text{ in})^2 \\ &\approx 200.96 \text{ in}^2 \end{aligned}$$

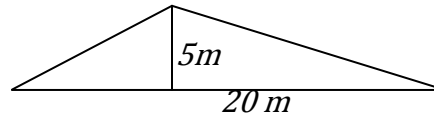


The area is exactly $64\pi \text{ in}^2$. The area is approximately 200.96 in^2 .

Area of a Triangle

In the figure below, AB is the base of the triangle and CD , which is perpendicular to the base, is the height of the triangle. The area of a triangle is one-half the product of the base and the height.

$$\begin{aligned} A &= \frac{1}{2} b h \\ &= \frac{1}{2} (20m)(5m) \\ &= 50 \text{ m}^2 \end{aligned}$$



The area of the triangle is 50 m^2 .

VOLUME

Volume is a measure of the amount of space inside a figure.

A cube that measures 1 ft on each side has a volume of 1 cubic foot, which is written 1 ft^3 .

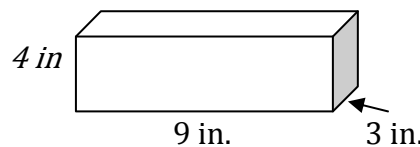
Volume of a Rectangular Solid

The volume of a solid is the number of cubes that are necessary to fill the solid exactly.

$$V = LWH$$

Find the volume of a rectangular solid with a length of 9 in., a width of 3 in., and a height of 4 in.

$$\begin{aligned} V &= LWH \\ &= (9 \text{ in}) (3 \text{ in}) (4 \text{ in}) \\ &= 108 \text{ in}^3 \end{aligned}$$



The volume of the rectangular solid is 108 in^3 .

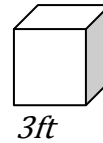
Volume of a Cube

The length, width, and height of a cube have the same measure; therefore, the volume of a cube is found by multiplying the side of the cube times itself three times (side cubed).

$$V = s^3$$

Find the volume of the cube shown at the right.

$$\begin{aligned} V &= s^3 \\ &= (3 \text{ ft})^3 \\ &= 27 \text{ ft}^3 \end{aligned}$$



The volume of the cube is 27 ft^3 .

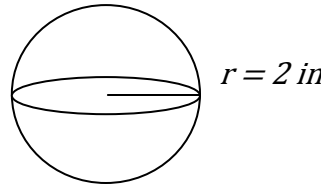
Volume of a Sphere

The volume of a sphere is found by multiplying four-thirds times pi (π) times the radius cubed.

$$V = \frac{4}{3} \pi r^3$$

Find the volume of the sphere shown below. Use 3.14 for π . Round to the nearest hundredth.

$$\begin{aligned} V &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \pi (2 \text{ in})^3 \\ &\approx \frac{4}{3} (3.14)(8 \text{ in}^3) \\ &\approx 33.49 \text{ in}^3 \end{aligned}$$



The volume is approximately 33.49 in^3 .

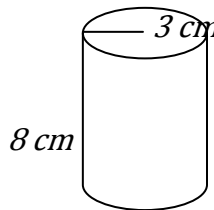
Volume of a Cylinder

The volume of a cylinder is found by multiplying the area of the base of the cylinder (a circle) times the height.

$$V = \pi r^2 h$$

Find the volume of the cylinder shown.

$$\begin{aligned} V &= \pi r^2 h \\ &\approx 3.14 (3 \text{ cm})^2 (8 \text{ cm}) \\ &\approx 3.14 (9 \text{ cm}^2)(8 \text{ cm}) \\ &\approx 226.08 \text{ cm}^3 \end{aligned}$$



The volume of the cylinder is approximately 226.08 cm^3 .

Volume of a Composite Geometric Solid

Composite geometric solids are solids made from two or more geometric solids.



The solid shown is made from a cylinder and one-half of a sphere.

The volume of the composite solid = the volume of the cylinder + $\frac{1}{2}$ volume of the sphere.

Find the volume of the composite solid shown above if the radius of the base of the cylinder is 3 in. and the height of the cylinder is 10 in. Use 3.14 for π .

$$\begin{aligned} V &= \pi r^2 h + \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right) \\ &\approx 3.14 (3 \text{ in})^2 (10 \text{ in}) + \frac{1}{2} \left(\frac{4}{3} \right) (3.14)(3 \text{ in})^3 \\ &\approx 3.14 (9 \text{ in}^2)(10 \text{ in}) + \frac{1}{2} \left(\frac{4}{3} \right) (3.14)(27 \text{ in}^3) \\ &\approx 282.6 \text{ in}^3 + 56.52 \text{ in}^3 \\ &\approx 339.12 \text{ in}^3 \end{aligned}$$

The volume is approximately 339.12 in^3 .

PRACTICE PROBLEMS

Set 1

1. Convert 63 ft to yards.
2. Subtract: $4\frac{2}{5} \text{ mi} - 3\frac{7}{10} \text{ mi}$
3. How long must a board be if four pieces, each 2 ft. 5 in. long, are cut from the board?
4. Convert: 50 oz = _____ lb _____ oz
5. Convert 7600 lb to tons
6. Multiply: 3 lb 5 oz \times 9
7. A baby weighed 5 lb 6 oz at birth. Two months later, the baby weighed 9 lb 2 oz. What was the baby's weight gain over the 2 months?
8. Convert $2\frac{1}{2}$ qt to pints.
9. Find the sum of 9 c 3 fl oz and 3 c 9 fl oz.
10. If 45 people are expected to attend a party, how many quarts of punch should be prepared if each person is expected to drink 3 c?

Set 2

1. Multiply: 3 ft 4 in. \times 5
2. Two boards that are $1\frac{3}{8}$ in. thick and $1\frac{2}{3}$ in. thick are nailed together. How thick is the resulting board?
3. You use 27 in. of ribbon off of a roll of ribbon containing 2 yd. How many inches of ribbon are left on the roll?
4. Convert 40 oz to pounds.
5. Divide: (4 tons 400 lb) \div 3
6. A case of canned goods contains 24 18-ounce cans. What is the weight in pounds of the case?
7. Subtract: 5 gal 1 qt - 3 gal 2 qt
8. Add: 3 qt 1 pt + 5 qt 1 pt
9. If a coffee mug contains 12 fl oz, how many mugs can be filled from a 6-quart container?
10. If 1 qt of paint will cover 80 sq ft, how many gallons of paint will be required to paint a room with 320 sq ft of wall space?

Set 3

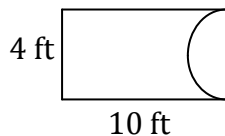
1. Convert 76.5 km to meters
2. Convert 3 m 5 cm to centimeters
3. A hiker can walk 7750 m in 1 hour. How far can the hiker walk (in kilometers) in 3 hours?
4. Convert 0.023 g to milligrams
5. Convert 5 kg 760 g to kilograms
6. Find the cost of a beef roast weighing 6700 g if the cost per kilogram is \$5.24. Round to the nearest cent.
7. Convert 130 cm^3 to milliliters
8. Convert 37 mL to liters.
9. Convert 2 L 65 mL to liters.
10. One hundred thirty people are going to attend a wedding reception. Assuming that each person drinks 400 mL of punch, how many liters of punch should be purchased?

Set 4

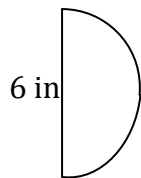
1. Convert 23.2 km to meters
2. Convert 432 mm to centimeters
3. Convert 7 m 89 cm to meters
4. A rectangular garden measures 975 cm long and 625 cm wide. What total length of fencing (in meters) is required to enclose the field?
5. Convert 27 mg to grams
6. Convert 2 kg 470 g to kilograms
7. A brick weighs 985 g. Find the weight (in kilograms) of a load of 200 bricks.
8. Convert 36 ml to cubic centimeters
9. Convert 5 L 294 ml to liters.
10. There are 48 bottles in a case of shampoo. Each bottle contains 250 ml of shampoo. How many liters of shampoo are in 1 case?

Set 5

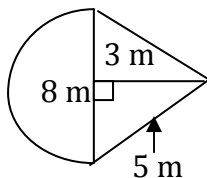
1. Find the circumference of a circle with a radius of 8 cm. Use 3.14 for π .
2. Find the perimeter of the composite figure below. Use 3.14 for π .



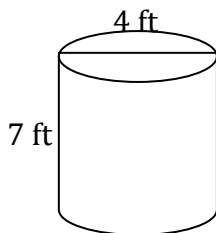
3. A homeowner is putting molding around the ceiling of a room that is 14 ft wide and 22 ft long. At \$2.35 per foot, how much will it cost for the molding?
4. Find the area of a right triangle with a base of 4 ft and a height of $\frac{2}{3}$ ft.
5. Find the area of the composite figure below. Use 3.14 for π .



6. Find the area of the composite figure below. Use 3.14 for π .



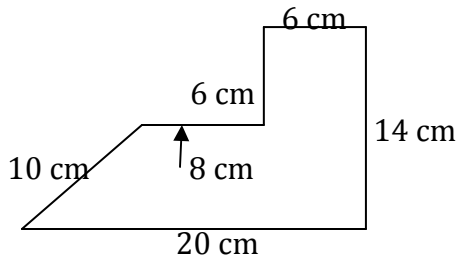
7. A room 12 ft by 16 ft is carpeted. At \$18 per square yard, how much does it cost to carpet the room? ($9 \text{ ft}^2 = 1 \text{ yd}^2$)
8. Find the volume of a cube with a side of $3\frac{1}{2}$ in.
9. Find the volume of the solid cylinder shown below. Use 3.14 for π .



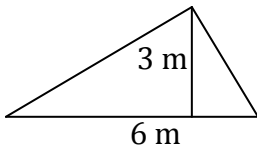
10. How many gallons of water will fill a fish tank that is 18 in. long, 10 in. wide, and 9 in. high? Round to the nearest tenth. ($1 \text{ gal} = 231 \text{ in}^3$)

Set 6

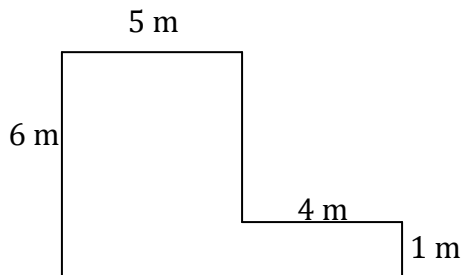
1. Find the perimeter of a square in which the sides are equal to 1 ft 4 in.
2. Find the perimeter of the composite figure shown below.



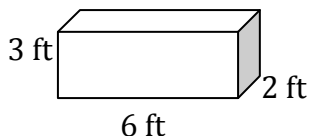
3. Find the number of feet of framing needed to frame a picture that is $3\frac{1}{3}$ by $2\frac{1}{2}$ ft.
4. Find the area of a square with a side of 3.1 m.
5. Find the area of the triangle shown below.



6. A carpet is to be placed in a room and hallway, as shown in the diagram below. At \$18.50 per square meter, how much will it cost to carpet the area?



7. Two rooms, one 12 ft by 16 ft and the other 18 ft by 24 ft, are to be carpeted. Find the total number of square yards of carpet needed. Round to the nearest tenth. ($9\text{ ft}^2 = 1\text{ yd}^2$)
8. Find the volume of a sphere with a 6-centimeter radius. Use 3.14 for π .
9. Find the volume of the rectangular solid shown below.



10. An oil storage tank that is in the form of a cylinder is 15 ft high and has a 30-foot diameter. Find the volume of the storage tank. Use 3.14 for π .

ANSWERS TO PRACTICE PROBLEMS

Set 1

- 21 yd
- $\frac{7}{10}$ mi
- 9 ft 8 in
- 3 lb 2 oz
- $3\frac{4}{5}$ tons
- 29 lb 13 oz
- 3 lb 12 oz
- 5 pt
- 13 c 4 fl oz
- $33\frac{3}{4}$ qt

Set 2

- 16 ft 8 in
- $3\frac{1}{24}$ in
- 45 in
- $2\frac{1}{2}$ lb
- 1 ton 800 lb
- 27 lb
- 1 gal 3 qt
- 9 qt
- 16 mugs
- 1 gallon

Set 3

- 76,500 m
- 305 cm
- 23.250 km
- 23 mg
- 5.760 kg
- \$35.11
- 130 ml
- 0.037 L
- 2.065 L
- 12 L

Set 4

- 23,200 m
- 43.2 cm
- 7.89 m
- 32 m
- 0.027 g
- 2.470 kg
- 197 kg
- 36 cm^3
- 5.294 L
- 12 L

Set 5

- 50.24 cm
- 30.28 ft
- \$169.20
- $1\frac{1}{3}\text{ ft}^2$
- 14.13 in^2
- 37.12 m^2
- \$384
- $42\frac{7}{8}\text{ in}^3$
- 87.92 ft^3
- 7.0 gal

Set 6

- 5 ft 4 in
- 64 cm
- $11\frac{2}{3}\text{ ft}$
- 9.61 m^2
- 9 m^2
- \$629
- 69.3 yd^2
- 904.32 cm^3
- 36 ft^3
- $10,597.5\text{ ft}^3$