Introduction to Decimals

Reading and Writing Decimals:

Note: There is a relationship between fractions and numbers written in decimal notation.

Three-tenths	Three-hundredths	Three-thousandths
$\dfrac{3}{10} = 0.\underline{3}$	$\dfrac{3}{100} = 0.\underline{03}$	$\dfrac{3}{1000} = 0.\underline{003}$
1 zero 1 decimal place	2 zeros 2 decimal places	3 zeros 3 decimal places

A number written in decimal notation has three parts.

351 **.** 7089

Whole-number part   Decimal Point   Decimal Part

A number written in decimal notation is often called simply a **decimal**. The position of a digit in a decimal determines the digit's place value.

In the decimal 6,584,791.0324, the position of the digit 2 determines that its place value is thousandths.

When writing a decimal in words, write the decimal part as if it were a whole number; then name the place value of the last digit.

0.6481   Six thousand four hundred eighty-one ten-thousandths
549.238   Five hundred forty-nine **and** two hundred thirty-eight thousandths
            (The decimal point is read as "and")

To write a decimal in standard form, zeros may have to be inserted after the decimal point so that the last digit is in the given place-value position.

Five and thirty-eight <u>hundredths</u>   5.3<u>8</u>
                                    (8 is in the hundredths' place.)

Nineteen and four <u>thousandths</u>   19.00<u>4</u>
                        (Insert two zeros so that the 4 is in the thousandths' place.)

Seventy-one <u>ten-thousandths</u>   0.007<u>1</u>
                        (Insert two zeros so that 1 is in the ten-thousandths' place.)

Example 1   Write 307.4027 in words.

Solution   Three hundred seven and four thousand twenty-seven ten-thousandths

Example 2   Write six hundred seven and seven hundred eight hundred-thousandths in standard form.

Solution   607.00708

Rounding Decimals:

Rounding decimals is similar to rounding whole numbers except that the digits to the right of the given place value are dropped instead of being replaced by zeros.

If the digit to the right of the given place value is less than 5, drop the digit and all digits to the right. If the digit to the right of the given place value is greater than or equal to 5, increase the number in the given place value by 1 and drop all digits to its right.
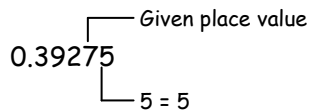
➡ Round 26.3799 to the nearest hundredth.

26.3799   Given place value

     9 > 5   Increase 7 by 1 and drop all digits to the right of 7.

26.3799 rounded to the nearest hundredth is 26.38.
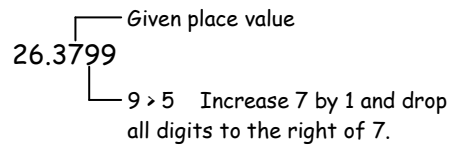
Example 3   Round 0.39275 to the nearest ten-thousandth.

Solution

Given place value

0.39275

     5 = 5

0.3928

Example 4   Round 42.0237412 to the nearest hundred-thousandth.

Solution

Given place value

42.0237412

     1 < 5
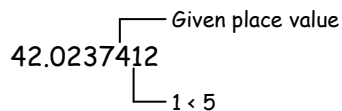
42.02374

Addition and Subtraction of Decimals

To add or subtract decimals, write the numbers so that the decimal points are on a vertical line, placing zeros as place holders if necessary. Add or subtract as for whole numbers, and write the decimal point in the sum or difference directly below the decimal points in the addends or subtrahends.

➡ Add: $0.237 + 4.9 + 27.32$

$$
\begin{array}{r}
0.237 \\
4.900 \\
+27.320 \\
\hline
32.457
\end{array}
$$

➡ Subtract: $21.532 - 9.875$

$$
\begin{array}{r}
21.532 \\
-9.875 \\
\hline
11.657
\end{array}
$$

➡ Subtract: $4.3 - 1.7942$

$$
\begin{array}{r}
4.3000 \\
-1.7942 \\
\hline
2.5058
\end{array}
$$

Note: By placing the decimal points on a vertical line, we make sure that digits of the same place value are added or subtracted.

Multiplication of Decimals

To multiply decimals, multiply the numbers as in whole numbers. Write the decimal point in the product so that the number of decimal places in the product is the sum of the decimal places in the factors.

➡ Multiply: 21.4×0.36

$$\begin{array}{r} 21.4 \\ \times\, 0.36 \\ \hline 1284 \\ 642 \\ \hline 7.704 \end{array}$$

21.4   1 decimal place
× 0.36   2 decimal places
1284
642
7.704   3 decimal places

➡ Multiply: 0.037×0.08

$$\begin{array}{r} 0.037 \\ \times\, 0.08 \\ \hline 0.00296 \end{array}$$

0.037   3 decimal places
× 0.08   2 decimal places
0.00296   5 decimal places

Note: Two zeros must be inserted between the 2 and the decimal point so that there are 5 decimal places in the product.

Multiplying by Powers of 10:

To multiply a decimal by a power of 10 (10, 100, 1000, …), move the decimal point to the right the same number of places as there are zeros in the power of 10.

$3.8925 \times 10 = 38.925$ $3.8925 \times 10^1 = 38.925$

$3.8925 \times 100 = 389.25$ $3.8925 \times 10^2 = 389.25$

$3.8925 \times 1000 = 3892.5$ $3.8925 \times 10^3 = 3892.5$

$3.8925 \times 10,000 = 38,925.$ $3.8925 \times 10^4 = 38,925.$

$3.8925 \times 100,000 = 389,250.$ $3.8925 \times 10^5 = 389,250.$

Note: If a power of 10 is written in exponential notation, the exponent indicates how many places to move the decimal point.

Division of Decimals

To divide decimals, move the decimal point in the divisor to the right to make the divisor a whole number. Move the decimal point in the dividend the same number of places to the right. Place the decimal point in the quotient directly over the decimal point in the dividend, and then divide as in whole numbers.

Divide: $3.25\overline{)15.275}$

Move the decimal point 2 places to the right in the divisor and then in the dividend. Place the decimal point in the quotient.

$$\begin{array}{r} 4.7 \\ 325\overline{)1527.5} \\ -1300 \\ \hline 227\,5 \\ -227\,5 \\ \hline 0 \end{array}$$

Moving the decimal point the same number of decimal places in the divisor and dividend does not change the value of the quotient, because this process is the same as multiplying the numerator and denominator of a fraction by the same number. In the example above,

$$3.25\overline{)15.275} = \frac{15.275}{3.25} = \frac{15.275\times100}{3.25\times100} = \frac{1527.5}{325} = 325\overline{)1527.5}$$

Dividing by Powers of 10:

To divide by powers of 10 (10, 100, 1000, …), move the decimal point to the left the same number of places as there are zeros in the power of 10.

$34.65 \div 10 = 3.465$ $34.65 \div 10^1 = 3.465$

$34.65 \div 100 = 0.3465$ $34.65 \div 10^2 = 0.3465$

$34.65 \div \underline{1000} = 0.03465$ $\qquad$ $34.65 \div 10^3 = 0.03465$

$34.65 \div \underline{10{,}000} = 0.003465$ $\qquad$ $34.65 \div 10^4 = 0.003465$

Note: If the power of 10 is written in exponential notation, the exponent indicates how many places to move the decimal point.

Comparing and Converting Fractions and Decimals

Convert Fractions to Decimals:

Every fraction can be written as a decimal. To write a fraction as a decimal, divide the numerator of the fraction by the denominator. The quotient can be rounded to the desired place value.

➡ Convert $\frac{3}{7}$ to decimal.

$$\begin{array}{r} 0.42857 \\ 7\overline{)3.00000} \end{array}$$ $\qquad$ $\frac{3}{7}$ rounded to the nearest hundredth is 0.43.

$\frac{3}{7}$ rounded to the nearest thousandth is 0.429.

$\frac{3}{7}$ rounded to the nearest ten-thousandth is 0.4286.

➡ Convert $3\frac{2}{9}$ to a decimal. Round to the nearest thousandth.

$$3\frac{2}{9} = \frac{29}{9} \qquad \begin{array}{r} 3.2222 \\ 9\overline{)29.0000} \end{array} \qquad 3\frac{2}{9} \text{ rounded to the nearest thousandth is 3.222.}$$

Convert Decimals to Fractions:

To convert a decimal to a fraction, remove the decimal point and place the decimal part over a denominator equal to the place value of the last digit in the decimal. Reduce if possible.

hundredths
$$0.47 = \frac{47}{100}$$

thousandths
$$0.275 = \frac{275}{1000} = \frac{11}{40}$$

hundredths
$$7.45 = 7\frac{45}{100} = 7\frac{9}{20}$$

hundredths
$$0.16\frac{2}{3} = \frac{16\frac{2}{3}}{100} = 16\frac{2}{3} \div 100 = \frac{50}{3} \times \frac{1}{100} = \frac{1}{6}$$

Identify Order Relations Between 2 Decimals or a Decimal & a Fraction:

Decimals, like whole numbers and fractions, can be graphed as points on the number line. The number line can be used to show the order of decimals. A decimal that appears to the right of a given number is greater than the given number. A decimal that appears to the left of a given number is less than the given number.
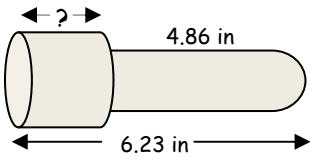

3.00 3.05 3.10 3.15 3.20 3.25 3.30 3.35 3.40
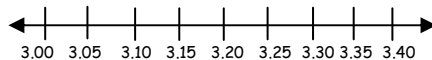
Note: 3, 3.0, and 3.00 represent the same number.

➡ Find the order relation between $\frac{3}{8}$ and 0.38.

$$\frac{3}{8} = 0.375 \quad 0.38 = 0.380$$
$$0.375 < 0.380$$
$$\frac{3}{8} < 0.38$$

Practice Problems

1. Find the quotient of 3.6515 and 0.067.
2. Find the sum of 369.41, 88.3, 9.774, and 366.474.
3. Place the correct symbol, $<$ or $>$, between the two numbers.   0.055   0.1
4. Write 22.0092 in words.
5. Round 0.05678235 to the nearest hundred-thousandth.
6. Convert $2\frac{1}{3}$ to a decimal. Round to the nearest hundredth.
7. Convert 0.375 to a fraction.
8. Add: $3.42 + 0.794 + 32.5$
9. Write thirty-four and twenty-five thousandths in standard form.
10. Place the correct symbol, $<$ or $>$, between the two numbers. $\frac{5}{8}$ 0.62

11. Convert $\frac{7}{9}$ to a decimal. Round to the nearest thousandth.
12. Convert $0.66\frac{2}{3}$ to a fraction.
13. Subtract: $27.31 - 4.4465$
14. Round 7.93704 to the nearest hundredth.
15. Find the product of 3.08 and 2.9.
16. Write 342.37 in words.
17. Write three and six thousand seven hundred fifty-three hundred-thousandths in standard form.
18. Multiply: 34.79×0.74
19. Divide: $0.053\overline{)0.349482}$
20. What is 7.796 decreased by 2.9175?

21. You had a balance of $895.68 in your checking account. You then wrote checks of $145.72 and $88.45. Find the new balance in your checking account.
22. The state income tax on the business you own is $560 plus 0.08 times your profit. You made a profit of $63,000 last year. Find the amount of income tax you paid last year.
23. A car costing $5944.20 is bought with a down payment of $1500 and 36 equal monthly payments. Find the amount of each monthly payment.
24. You have $237.44 in your checking account. You make deposits of $56.88, $127.40, and $56.30. Find the amount in your checking account after you make the deposits.

25. Place the correct symbol, $<$ or $>$, between the two numbers. 0.66   0.666
26. Subtract: $13.027 - 8.94$
27. Write 45.0302 in words.
28. Convert $\frac{9}{13}$ to a decimal. Round to the nearest thousandth.
29. Convert 0.825 to a fraction.
30. Round 0.07395 to the nearest ten-thousandth.
31. Find 0.0569 divided by 0.037. Round to the nearest thousandth.
32. Find 9.23674 less than 37.003.

33. Round 7.0954625 to the nearest thousandth.
34. Divide: $0.006\overline{)1.392}$
35. Add: $270.93 + 97 + 1.976 + 88.675$
36. Find the missing dimension.

37. Multiply: 1.37×0.004
38. What is the total of 62.3, 4.007, and 189.65?

39. Write two hundred nine and seven thousand eighty-six hundred-thousandths in standard form.
40. A car was bought for $6392.60, with a down payment of $1250. The balance was paid in 36 monthly payments. Find the amount of each monthly payment.
41. You received a salary of $363.75, a commission of $954.82, and a bonus of $225. Find your total income.
42. A long-distance telephone call costs $.85 for the first 3 minutes and $.42 for each additional minute. Find the cost of a 12-minute long-distance telephone call.

The table below shows the postal rates for selected weights for single-piece and presorted mail. Use this table for Exercises 43 and 44.

Weight (not over)	Single piece	Presorted
1 ounce	$.32	$.295
2 ounces	.55	.525
3 ounces	.78	.709
4 ounces	1.01	.939
5 ounces	1.24	1.69

43. Find the total cost of mailing the following: 3 single pieces weighing 2.3 ounces each, 2 single pieces weighing 3.4 ounces each, and 50 presorted pieces weighing 0.3 ounce each.

44. Find the total cost of mailing the following: 7 single pieces weighing 0.4 ounce each, 30 presorted pieces weighing 0.7 ounce each, and 75 presorted pieces weighing 1.4 ounces each. Round to the nearest cent.

45. Write 129.04 in words.

46. Write in standard form: six and two thousand fifty-three ten-thousandths.

47. Round 47.28351 to the nearest thousandth.

48. Round 3792.6974 to the nearest hundredth.

49. Add: $42.97 + 162.9 + 12.876 + 1.093$

50. Find the sum of 8962.51, 54.3, and 123.017.

51. You have $692.46 in your checking account. You make deposits of $41.30, $122.79, and $98.42. You write a check for $219.67. Find the amount in your checking account after making the deposits and writing the check.

52. Subtract: $20.092 - 17.9263$

53. Find 25.367 less than 108.44.

54. An electrician earns $3129.60. Deductions from the check are $630.24 for federal tax, $180.92 for state tax, and $43.12 for insurance. Find the electrician's take-home pay.

55. Multiply: 8.35×0.017

56. Find the product of 0.93 and 0.027.

57. A car travels 32.3 miles on each gallon of gas. How far does a car travel on 11.8 gallons of gas? Round to the nearest tenth of a mile.

58. Find 2.192 divided by 0.004.

59. Divide: $34\overline{)5.072}$ Round to the nearest thousandth.

60. You work 40 hours and receive $302. Find your hourly rate of pay.

61. Convert $\frac{7}{18}$ to a decimal. Round to the nearest thousandth.

62. Convert 0.025 to a fraction.

63. Place the correct symbol, $<$ or $>$, between the two numbers.   0.017   0.17

64. Place the correct symbol, $<$ or $>$, between the two numbers. $\frac{5}{8}$ 0.6255

65. Write 4.00632 in words.

66. Write twelve and six hundredths in standard form.

67. Round 37.295 to the nearest hundredth.

68. Round 43.0973 to the nearest tenth.

69. Add: $69.94 + 9.807 + 994.673 + 95.06$

70. Find the total of 34.92, 6.817, and 152.9.

71. Kara Brady bought a math textbook for $39.43, a history text for $43.95, a biology text for $35.61, and an English workbook for $19.45. Find the total cost of the four books.

72. Subtract: $109.63 - 94.984$

73. What is 84.5903 decreased by 9.953?

74. Your lunch costs $4.87. An additional cost of $.29 is added for tax. Find the change you receive from a $10 bill.

75. Multiply: 0.086×0.07

76. Find the product of 16.5 and 0.24

77. A motorist averages 56.4 miles per hour while traveling for 3.5 hours. Find the total distance traveled during the 3.5 hours.

78. Find the quotient of 0.042 and 14.

79. Divide: $0.076\overline{)0.02904}$ Round to the nearest thousandth.

80. A $563.68 Sony stereo system is bought for a down payment of $100 and 18 equal monthly payments. Find the amount of each monthly payment.

81. Convert $\frac{9}{40}$ to a decimal.

82. Convert $0.16\frac{2}{3}$ to a fraction.

83. Place the correct symbol, $<$ or $>$, between the two numbers.   0.72   0.702

84. Place the correct symbol, $<$ or $>$, between the two numbers. $\frac{17}{24}$ 0.7

Practice Problems Key

1. 54.5
2. 833.958
3. $0.055 < 0.1$
4. Twenty-two and ninety-two ten-thousandths
5. 0.05678
6. 2.33
7. $\frac{3}{8}$
8. 36.714
9. 34.025
10. $\frac{5}{8} > 0.62$
11. 0.778
12. $\frac{2}{3}$
13. 22.8635
14. 7.94
15. 8.932
16. Three hundred forty-two and thirty-seven hundredths
17. 3.06753
18. 25.7446
19. 6.594
20. 4.8785
21. $661.51
22. $5600.
23. $123.45/month
24. $478.02
25. $0.66 < 0.666$
26. 4.087
27. Forty-five and three hundred two ten-thousandths
28. 0.692
29. $\frac{33}{40}$
30. 0.0740
31. 1.538
32. 27.76626
33. 7.095
34. 232
35. 458.581
36. 1.37 inches
37. 0.00548
38. 255.957
39. 209.07086
40. $142.85/month
41. $1543.57
42. $4.63
43. $19.11
44. $50.47
45. one hundred twenty-nine and four hundredths
46. 6.2053
47. 47.284
48. 3792.70
49. 219.839
50. 9139.827
51. $735.30
52. 2.1657
53. 83.073
54. $2275.32
55. 0.14195
56. 0.02511
57. 381.1 miles
58. 548
59. 0.149
60. $7.55/hour
61. 0.389
62. $\frac{1}{40}$
63. $<$
64. $<$
65. four and six hundred thirty-two hundred-thousandths
66. 12.06
67. 37.30
68. 43.1
69. 1169.48
70. 194.637
71. $138.44
72. 14.646
73. 74.6373
74. $4.84
75. 0.00602
76. 3.96
77. 197.4 miles
78. 0.003
79. 0.382
80. $25.76/month
81. 0.225
82. $\frac{1}{6}$
83. $>$
84. $>$