Rules of Algebra

Quadratic Formula

If a ≠ 0, the roots of

$$ax^2 + bx + c = 0$$

are

$$x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

Special Product Formulas

$$(x+y)(x-y) = x^2 - y^2$$
$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$
$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

Special Factoring Formulas

$$x^2 - y^2 = (x+y)(x-y)$$
$$x^2 + 2xy + y^2 = (x+y)^2$$
$$x^2 - 2xy + y^2 = (x-y)^2$$
$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$
$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

Exponents and Radicals

$$a^m a^n = a^{m+n}a \qquad a^{1/n} = \sqrt[n]{a}$$
$$(a^m)^n = a^{mn} \qquad a^{m/n} = \left(\sqrt[n]{a}\right)^m$$
$$(ab)^n = a^n b^n \qquad a^{m/n} = \sqrt[n]{a^m}$$
$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad \sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$$
$$\frac{a^m}{a^n} = a^{m-n} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$
$$a^{-n} = \frac{1}{a^n} \qquad \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

Binomial Theorem

$$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \cdots + \binom{n}{k}x^{n-k}y^k + \cdots + y^n$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Inequalities

If a > b and b > c, then a > c.

If a > b, then a+c > b+c.

If a > b and c > 0, then ac > bc.

If a > b and c < 0, then ac < bc.

Absolute Value (p > 0)

$|x| = p$ if and only if either $x = p$ or $x = -p$, which means $x = \mp p$

$|x| < p$ if and only if both $x < p$ and $x > -p$, which means $-p < x < p$.

$|x| \le p$ if and only if both $x \le p$ and $x \ge -p$, which means $-p \le x \le p$.

$|x| > p$ if and only if either $x > p$ or $x < -p$.

$|x| \ge p$ if and only if either $x \ge p$ or $x \le -p$.

Sequences and Series

Sum S_n of the first *n* terms of an arithmetic sequence with first term a_1 and common difference *d*

$$S_n = \tfrac{n}{2}(a_1 + a_2) \text{ or } S_n = \tfrac{n}{2}[2a_1 + (n-1)d]$$

Sum S_n of the first *n* terms of a geometric sequence with first term a_1 and common ration *r*

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad \text{or} \quad S_\infty = \frac{a_1}{1-r}$$

Arithmetic mean *A* of *n* numbers

$$A = \frac{a_1 + a_2 + \cdots + a_n}{n}$$

Geometric mean *G* of *n* numbers

$$G = \sqrt[n]{a_1 a_2 \ldots a_n}, \qquad a_k > 0$$

Exponentials and Logarithms

$$y = \log_a x \quad \textit{means} \quad a^y = x$$
$$\log_a xy = \log_a x + \log_a y$$
$$\log_a \frac{x}{y} = \log_a x - \log_a y$$
$$\log_a x^r = r \log_a x$$
$$a^{\log_a x} = x$$
$$\log_a a^x = x$$
$$\log_a 1 = 0$$
$$\log_a a = 1$$
$$\log x = \log_{10} x$$
$$\ln x = \log_e x$$
$$\log_b u = \frac{\log_a u}{\log_a b}$$